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AN OUTLINE OF PRINCALS

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INTRODUCTION

PRINCALS is a principal components technique, which includes generalizations to non-metric measurement levels, incomplete data matrices, and the analysis of categorical data discussed in De Leeuw & Van Rijckevorsel (1979). All data have to be discrete or discretized. The various measurement levels can be defined for each variable separately. Survey data, preference data, mixed data in general can be analyzed. PRINCALS can also be used as an ordination technique. Techniques and computer programs, based on similar ideas are PRINCIPALS (Young, De Leeuw & Takane, 1978); the non-metric factor analysis method of Kruskal & Shepard (1974); PRINQUAL (Tenenhaus, 1976); and HOMALS (De Leeuw, 1976), (Van Rijckevorsel & De Leeuw, 1978).

PRINCALS is also the name of a highly portable computer program, written in FORTRAN IV, to be used on computers of all makes. A complete user's guide with examples and output listing is provided in this paper. The algorithm is of the alternating least squares with optimal scaling type (Young, De Leeuw & Takane, 1979). Utmost attention is given to the efficiency, structure and clarity of the program.

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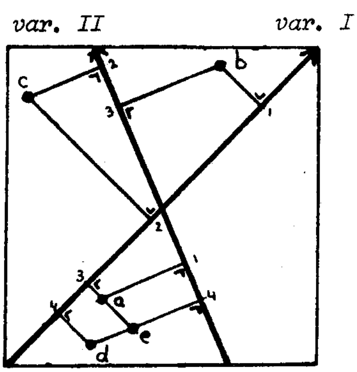
DATA REDUCTION AND GEOMETRY

We want to explain the variance of a larger set of numbers by a smaller set of numbers with nicer properties. This is called data reduction. This approach does not fit an explicit algebraic or stochastic model to the data by estimating free parameters but simply tries to replace a lot of unstructured input data by a smaller amount of possibly more structured output data.
The datamatrix is represented as a bundle of m vectors in a n-dim. space with the angles between them representing their relations. If the vectors are of unit length, these relations would be correlations equal to the cosines of the angles between them. It is clear that the m vectors always can be fitted into a m-dimensional space, since there are maximally m dimensions needed to represent m vectors. If there are m - p linearly independent directions in the m-dimensional subspace on which the original m vectors have zero projections, we can fit m variables in a p-dimensional subspace. We usually postulate a p considerably smaller than m and allow a certain distortion as we force the vector bundle into a p-space. The smaller p the more data reduction, but also the more distortion. The amount of distortion, which is minimized by the algorithm for given p is called stress or loss.

PERFECT FIT AND MEASUREMENT LEVELS

In the vector model the row entries or observations are represented as points, the column entries or variables as vectors and the possible values of the column entries (i.e. the categories of the variables) are perpendicular hyperplanes on the column vectors.



VARIABLES

OBSERVATIONS	I	II
a	3	1
b	1	3
c	2	2
d	4	4
e	3	4

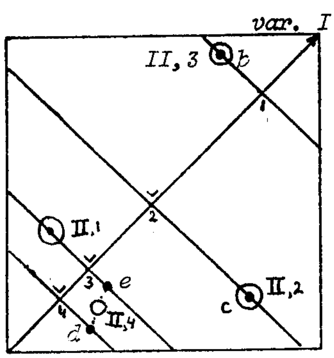
the corresponding data matrix

Figure 1

A possible representation of the vector model, illustrated for five observations and two variables, one ordinal (I) and the other nominal (II).

According to measurement level these parallel hyperplanes belonging to one variable have to be either ordered and equally spaced, which is the numerical option; or ordered only, which is the ordinal option; or not necessarily ordered, which is the single nominal option.

The nominal data approach in PRINCALS needs some explanation. PRINCALS does not only fit the vector model, but also the multiple PCA of categorical data of Guttman (1941), Burt (1950), Lingoes (1972) among others. This model is rediscovered by different authors about every ten years, most recently by Levine (1979). It is extensively discussed by De Leeuw (1973) and the most general computer program for this model is HOMALS by Van Rijckevorsel & De Leeuw (1978). The geometry of the multiple approach to nominal data is different from the single vector model in the way that no assumptions for a functional relationship within categories of a variable are made, while the vector model is actually fitting a straight line on which the category points are projected. The only restriction on category points in the multiple approach is that they have to be the centroids of points of observations that scored in those categories.



OBSERVATIONS		VARIABLES I	VARIABLES II
	a	3	1
	b	1	3
	c	2	2
	d	4	4
	e	3	4

Figure 2

Spatial representation of the ordinal single approach for one variable and the multiple nominal centroid approach for the other variable in case of perfect fit.

The multiple treatment produces a different category points configuration of the second variable, which is illustrated in figure 2. In comparing figure 1 and figure 2 one has to bear in mind that we kept the data constant and that only the treatment for the second variable is different. Category points in figure 2 coincide with observation points but only when there is just one observation is such a category; when there are more observations in a category the category point is the centroid of those observation points. cf. the point of category 4 of variable II in figure 2. Within the multiple approach category points are not necessarily on a vector anymore nor on corresponding hyperplanes.

Perfect fit is also different. In multiple PCA perfect fit means coincidence of category point and observation points of observations in that category. In single PCA observation points have to be on the hyperplanes of the categories in which they scored. In most practical situations perfect fit cannot be obtained and observation points will not coincide with their centroid but will be close, or, in the single approach, observation points will not be on the hyperplanes but close to them. Geometrically this means that the observation points, corresponding with a category, tend to be in small contiguous regions in the multiple approach, and in parallel strips in the single approach. See figure 3.

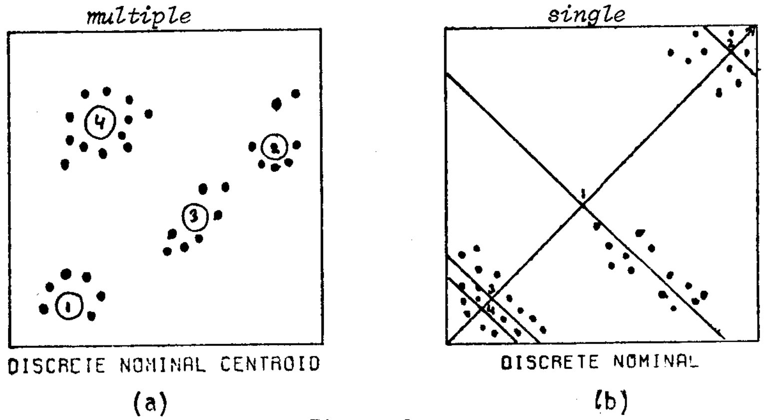


Figure 3

A spatial representation of the single approach (b) and the multiple approach (a) in case of non-perfect fit.

OTHER WORK

The idea of non-metric linear factor analysis is already proposed by Thurstone (1947). It was investigated theoretically by Coombs and Kao (1955) for the dichotomous case, by Bennett (1956) in the ordinal case and from the optimal scaling point of view by Guttman (1959). Actual computational methods have been proposed by Lingoes and Guttman (1967) with monotone regression of the scalar products of vectors on the covariances of the raw data, by Kruskal and Shepard (1974) with monotone regression of the projections of the observation points, on the raw data. Both techniques apply to ordinal data only, which is a drawback because the superiority of the ordinal approach above metric analysis is doubtful, even for " genuine " ordinal data (Woodward & Overall, 1976). Another method is proposed by Tenenhaus (1976); his program, called PRINQUAL, analyses nominal and numerical data with a generalized canonical analysis as the initial configuration and with a block relaxation algorithm.

The most recent alternative is PRINCIPALS by Young, De Leeuw & Takane (1978). They analyze mixed data, i.e. a mixture of nominal, ordinal and numerical data within one set, with optimal scaling of the transformed categories according to the measurement level of the raw data, either discrete or continuous, with an ALS algorithm and a singular value decomposition of the raw data as the initial configuration. Their method is limited by the relatively small size of problems that can be analyzed and by the computational inefficiency of the algorithm.
All these methods fit the linear vector model. PRINCALS resembles PRINCIPALS more than any other here mentioned technique, but there are still some differences. In the first place PRINCALS fits also the multiple approach for nominal data of Fischer (1940), Guttman (1941), Mosteller (1948), Hayashi (1950), or more recent Lingoes (1972), De Leeuw (1973), Nishisato (1978), Levine (1979), and thus HOMALS of De Leeuw (1976) and Van Rijckevorsel & De Leeuw (1978). This option gives a non-linear fit for variables and fits the vector model for categories of those variables; secondly, PRINCALS analyses only discrete or discretized data, which is the secondary approach to ties of Kruskal (1964); and thirdly, the quantifications in PRINCALS depend only on the non-missing entries. This means that PRINCALS is a generalized form of PCA of incomplete matrices. The normalizations are consequently weighted for the non-missing entries.
The controversy between metric- and non-metric PCA in the sense of optimal solutions is not solved by PRINCALS. Only the comparison between them is attractive by using PRINCALS because both solutions are always available on output.

INTERVAL DATA

One can define PCA in several ways. Here we prefer geometrical starting points and loss functions, derived from the geometry of the problem; that is, we prefer to see PCA as a multidimensional scaling method. Suppose we have n observations y_{ij} on m numerical variables ($i=1,\ldots,n$; $j=1,\ldots,m$). We want to represent these observations as points in a p-dimensional space and the variables as directions in that space i.e. as lines through the origin. Observation i is represented as the point $x_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$, and variable j is represented as the direction cosines $a_j = (a_{j1}, a_{j2}, \ldots, a_{jp})$. Thus $a_j'a_j = 1$ for all p. We require that the orthogonal projections of the observation points $x_1, x_2, \ldots, x_n$ on the direction defined by variable j are column-proportional with the measurement y_{ij}, $p \leqslant m$.

A perfect representation is thus defined by the requirements

$$y_{ij} = \Sigma_{s=1}^{p} x_{is} a_{js} \quad . \tag{1}$$

Clearly a perfect representation exists if and only if we can find a matrix X ($n \times p$) and a matrix A ($m \times p$) so that $Y = XA'$ i.e. if and only if rank $(Y) \leqslant p$. If a perfect solution does not exist for a chosen p we use the loss function:

$$\sigma = \mathrm{tr}\ (Y - XA')'(Y - XA') \qquad (2)$$

This loss function is to be minimized over X and A. The minimum can be found by means of a singular value decomposition of Y or by means of an eigenvalue-eigenvector decomposition of $Y'Y$ or YY'. These computational methods have two drawbacks. In the first place the computing time increases by the third power of the number of variables, which entails that really large datasets cannot be analyzed. Secondly these methods are not easily generalized to more general forms of PCA.

MISSING DATA

One of these more general forms of PCA is the case in which observations on several variables are missing. A suitable generalization of formula (2) is obtained by using diagonal $n \times n$ matrices M_j, indicating which observations on variable j are missing. Diagonal element i of M_j is unity if y_{ij} is *not* missing, and equals zero if y_{ij} is missing.and y_j is the j th column of Y. Hence we get:

$$\sigma_1 = \Sigma_{j=1}^{m}\ \mathrm{tr}\ (y_j - Xa_j)'M_j(y_j - Xa_j). \qquad (3)$$

This loss function has some serious disadvantages, of which the most important one is that minimization of (3) is not an eigenproblem anymore.
We consequently use an alternative:

$$\sigma_2 = \Sigma_{j=1}^{m}\ \mathrm{tr}\ (X - y_ja_j')'M_j(X - y_ja_j'). \qquad (4)$$

One has to prove that formula (4) is a generalization of formula (2). Suppose there are no missing data, i.e. $M_j = I$ for all j. Under the normalization requirements $X'X = I$ and $a_j'a_j = 1$ for all j we find:

$$\sigma_2 = \sigma_1 + m(p - 1). \qquad (5)$$

In this special case the minimization of σ_2 is equivalent to the minimization of σ_1. When $M_j \neq I$ for some j we have two different problems.

OPTIMAL SCALING

In the last two paragraphs we have assumed that all variables had at least an interval measurement level i.e. the elements y_{ij} are constants within the iterative process. We will generalize the idea by making the weaker assumption that the vectors vectors M_jy_j have to be in known convex cones K_j. The loss function in formula (4) has to be minimized over X, over A and over Y.. The minimization over Y for temporary fixed values of X and A during an iteration is sometimes called the *Optimal scaling* or the optimal quantification of variables (Young, De Leeuw and Takane, 1978). This so called *optimality* is defined in terms of a particular loss function. Every minimizing sub-operation within an iteration corresponds with a type of partitioning of the residual sum of squares. The particular optimal scaling partitioning is:

$$\sigma = \Sigma_{j=1}^{m} \text{tr } (a_j'a_j)(y_j - \hat{y}_j)'M_j(y_j - \hat{y}_j) + \Sigma_{j=1}^{m} \text{tr } M_jX\{I - \frac{a_ja_j'}{a_j'a_j}\} XM_j \text{ ,} \quad (6)$$

where

$$\hat{y}_j = \frac{1}{a_j'a_j} M_jXa_j \text{ .}$$

After some substitution we find

$$\sigma = \Sigma_{j=1}^{m} \text{tr } (a_j'a_j)(y_j'M_jy_j) - \Sigma_{j=1}^{m} \text{tr } 2y_j'M_jXa_j + \Sigma_{j=1}^{m} \text{tr } M_jXX'M_j \text{ ,} \quad (7)$$

which is evidently equal to formula (4) :

$$\sigma = \Sigma_{j=1}^{m} \text{tr } X'M_*X - \Sigma_{j=1}^{m} \text{tr } 2y_j'M_jXa_j + \Sigma_{j=1}^{m} \text{tr } a_jy_j'M_jy_ja_j' . \quad (8)$$

where

$$M_* = \Sigma_{j=1}^{m} M_j$$

We minimize the first term of (6) over y_j under the restriction that y_j is in the cone K_j. This defines a cone regression problem, the general theory of which is dealt with by De Leeuw (1977).

Metric PCA is the special case where the cones K_j are one-dimensional subspaces i.e. lines through the origin. In the case of ordinal variables the cones K_j are sets of isotonic or monotonic vectors i.e. those vectors on which the observations are ordered in the same way as on the original raw data vector. Because of the peculiar character of nominal variables we will deal with them separately.

NOMINAL VARIABLES

In our earlier paragraphs we have discussed a generalization of non-metric PCA with the according loss functions, a system of measurement levels, algorithms and geometrical representations. In the sequel we will deal with the question how nominal data do fit in this framework and what is the relationship between PRINCALS and the Guttman-Hayashi-Benzécri approach (= HOMALS) to categorical data.

nominal discrete data

Suppose a variable has k categories. We define k binary diagonal matrices M_c with the diagonal element M_{ci} equal to unity if observation i is in category c and equal to zero if i is not in category c. If there are no missing data the sum of all M_c is the identity matrix. The PRINCALS loss function for this variable is

$$\sigma = \Sigma_{c=1}^{k} \text{tr } (X - y_c a_c')' M_c (X - y_c a_c') \ . \qquad (9)$$

The optimal scaling restrictions for y_c are that all observations in category c will get the same score. So we treat our nominal variable with k categories as k binary variables with missing data. This is of course quite different from dividing a k-category variable into k complete binary variables. The relation with classical categorical PCA is rather straightforward (if no missing data exist). We can rewrite formula (4), using the (0,1) indicator matrix Δ with exactly one 1 in every row and the diagonal matrix of category frequencies $D = \Delta'\Delta$, as

$$\sigma = \text{tr } X'X - 2 \text{ tr } X'\Delta D^{-\frac{1}{2}} A + \text{tr } A'A \ . \qquad (10)$$

Define $\quad E = D^{-\frac{1}{2}}A.$

This implies

$$\sigma = \text{tr } (X - \Delta E)'(X - \Delta E). \qquad (11)$$

Now we can say that in case of no missing data and only nominal discrete variables, PRINCALS is equivalent to an eigen decomposition of the supermatrix C with submatrices

$$C_{j\ell} = D_j^{-\frac{1}{2}} \Delta_j' \Delta_\ell D_\ell^{-\frac{1}{2}} \ , \qquad (12)$$

which is the Guttman-Hayashi-Benzécri PCA for categorical data, a generalization of which is the computer program HOMALS of Van Rijckevorsel & De Leeuw (1978) and De Leeuw (1976). An extra advantage, which is not accidental, is that we can apply the geometrical properties of this form of PCA to our PRINCALS and HOMALS solutions. Formula (11) shows that perfect fit is defined as the coincidence of all observation points with their corresponding category points, and the minimization of (11) over all variables equals the minimization of the within-category variance of the representation, for further details see De Leeuw (1973, 1976).

THE ALGORITHM

The loss function in its basic form, i.e. $M_j = I$, for all j, is

$$\sigma = \Sigma_{j=1}^{m} \text{tr } (X - y_j a_j')'(X - y_j a_j'). \qquad (13)$$

The loss σ must be minimized over three sets of parameters y,a,X under the restrictions $X'X = I$, $a'a = 1$, and $y \in K$. We omit the variable subscripts because this is more convenient and it does not affect the results of this paragraph. The idea is that each substep of the algorithm minimizes the loss σ over one set for fixed values of the other two sets of parameters. Each iteration cycle consists of three substeps. In the beginning of an iteration we start with y^0, a^0, X^0. Each iteration gives three updates $y^\dagger$, $a^\dagger$, $X^\dagger$ each of which conditionally minimizes the loss σ.

Subproblems

To find the minimizing update $y^\dagger$ with a^0 and X^0 fixed, is a cone regression problem which has an unique solution (De Leeuw, 1977).
Finding an $a^\dagger$, with $y^\dagger$ and X^0 fixed, is an ordinary least squares problem with a unique solution. The component of σ depending on a is

$$\sigma\,(a) = \mathrm{tr}\,(X - ya')'(X - ya'), \tag{14}$$

$$= \mathrm{tr}\,X'X + \mathrm{tr}\,ay'ya' - 2\,\mathrm{tr}\,X'ya'\,, \tag{15}$$

$$= p + (a'a)(y'y) - 2\,a'X'y. \tag{16}$$

We have to minimize this over a satisfying $a'a = 1$. The solution is

$$a^\dagger = X^{0}{}'y^\dagger \,/\, (y^\dagger\; X^0X^{0}{}'y^\dagger)^{\frac{1}{2}} \tag{17}$$

Finding an $X^\dagger$ which minimizes σ with $a^\dagger$ and $y^\dagger$ under the requirement $X^\dagger{}'X^\dagger = I$ is an orthogonal procrustus problem, which has an unique solution if and only if the p-th singular value of the matrix $\frac{1}{m}\,\Sigma_{j=1}^{m}\, y_j^\dagger a_j^\dagger{}'$ is greater than the (p+1)-th singular value (Cliff,1966).

Properties of the algorithm

The loss function decreases except at a stationary point

Because all the suboperations are continuous operations the transformations

$$\begin{bmatrix} y^k \\ a^k \\ X^k \end{bmatrix} \rightarrow \begin{bmatrix} y^{k+1} \\ a^{k+1} \\ X^{k+1} \end{bmatrix}$$

are continuous.

Because $a'a = 1$, $X'X = I$ and $y'y = a'X'Xa \leqslant 1$ all our updates are in a compact set. We shall show further on that these three properties are sufficient to garantee the convergence of the algorithm.

For discrete variables it is useful to write y as

$$y = \Delta\hat{y}, \tag{18}$$

where Δ (n x k) is a (0,1) matrix, with in every row exactly one 1. We call such a matrix an indicator matrix (De Leeuw, 1973). $\hat{y}$ (k) is a vector with category numbers. We use this other notation because of the sparseness of the matrix Δ. The loss is now

$$\sigma = \mathrm{tr}\ (X - \Delta\hat{y}a')'M(X - \Delta\hat{y}a')\ . \tag{19}$$

This loss function is partitioned to show that minimization of one parameter-set with fixed values of the other parameters leads to new simpler quadratic loss functions. These more simple functions are relatively easy to minimize, which is a major advantage of the ALS algorithm.
The first partitioning with temporarily fixed values of $\hat{y}$ and a is

$$f\ (X|\hat{y},a) = f(\overline{X}) + \mathrm{tr}\ (X - \overline{X})'M(X - \overline{X}), \tag{20}$$

with

$$\overline{X} = \Delta\hat{y}a'. \tag{21}$$

This new quadratic function in X is minimized over all X satisfying

$$X'MX = I \tag{22}$$

The solution for X is discussed in the following paragraph.
The next partitioning with temporarily fixed values of X and $\hat{y}$ is

$$g\ (a|X,\hat{y}) = g(\overline{a}) + b(a - \overline{a})'(a - \overline{a}), \tag{23}$$

with

$$b = \hat{y}'\Delta'M\Delta\hat{y}, \tag{24}$$

and

$$\overline{a} = \frac{1}{b}\ X'M\Delta\hat{y}. \tag{25}$$

This is minimized over all a satisfying

$$a'a = 1. \tag{26}$$

The solution is

$$a = \overline{a}\ /\ (\overline{a}'\overline{a})^{\frac{1}{2}}. \tag{27}$$

The last partitioning with temporarily fixed values of X and a is

$$h(\hat{y}|X,a) = h(\bar{y}) + (\hat{y} - \bar{y})'D(\hat{y} - \bar{y}), \tag{28}$$

with

$$\bar{y} = D^{-1}\Delta'MXa, \tag{29}$$

and

$$D = \Delta'M\Delta. \tag{30}$$

This is minimized over all $\hat{y}$ satisfying $\hat{y} \in K$, where K is a convex cone.

Updating the matrix of observation scores

Minimizing (20) under the restrictions $X'MX = I$ and $e'MX = 0$ is equivalent to maximizing $\mathrm{tr}\ \bar{X}'MX$ with respect to X under the same restrictions. e is a vector of the appropriate length with unit elements. The observation scores matrix X is in deviation from the weighted mean. This second restriction prevents a trivial solution where all quantifications are constant. To meet the restrictions while maximizing $\mathrm{tr}\ \bar{X}'MX$, we use Lagrange multipliers S, a symmetric matrix, for $X'MX$ and t, a vector, for $e'MX$.

$$\sigma_{\max.\ X} = \mathrm{tr}\ \bar{X}'MX - \mathrm{tr}\ (X'MX - I)S - \mathrm{tr}\ t'X'Me. \tag{31}$$

We differentiate σ for X and set the partial derivative equal to the zero vector and if we define

$$J = M - \frac{Mee'M}{e'Me}, \tag{32}$$

and

$$A = J\bar{X} \tag{33}$$

the solution turns out to be

$$X = M^{-1}A(A'M^{-1}A)^{-\frac{1}{2}} \tag{34}$$

Computing the right hand term of (34) is equivalent to finding an orthonormal basis for the columnspace of $M^{-\frac{1}{2}}A$. This can be solved by a family of decomposition techniques like the singular value decomposition (SVD) of $M^{-\frac{1}{2}}A$, or by computing the eigenstructure of $A'MA$, or by the procrustean orthogonalization of $M^{-\frac{1}{2}}A$,

$$M^{-\frac{1}{2}}A = ST, \tag{35}$$

where T is a symmetric matrix and

$$S'S = I. \tag{36}$$

To compute (35) one can use a class of iterative methods proposed by Leipnik (1971). These methods are more efficient than the first two mentioned techniques. A fourth way to obtain the required basis is the modified Gram-Schmidt orthogonalization (MGS), which entails a stepwise orthonormalization of the columnspace of $M^{-\frac{1}{2}}A$ (Stewart,1973),

$$M^{-\frac{1}{2}}A = LR, \tag{37}$$

where L is a upper triangular matrix and

$$R'R = I. \tag{38}$$

This matrix L is a rotation of the matrix S,

$$S = LU, \tag{39}$$

with

$$U'U = UU' = I. \tag{40}$$

The total loss, when using MGS, is equal to the total loss obtained by the earlier mentioned techniques. The separate loss components are different however. The update for X is a rotation of the procrustean update and the corresponding additional loss is compensated for by the smaller loss components for the updates of a and y. The final updates are not affected because after ' convergence' the final solution is rotated to principal components and all quantifications are recomputed afterwards.

The stress and fit

The loss σ can be partitioned into several components. The total loss is called stress. The first component σ_1 due to the multiple fit of the data, is called multiple stress; the second component σ_2 , due to the fit of the single vector model, is called single stress. This single stress has also two components: σ_{2a} of the fit of the direction cosines and σ_{2b} of the fit of the optimally scaled vectors.

Multiple fit. Define $E = D^{-1}\Delta'MX$ and substitute

$$\hat{y}a' = E + (\hat{y}a' - E), \tag{41}$$

in (19)

$$\sigma = tr\,(X - \Delta E)'M(X - \Delta E) + tr(E - \hat{y}a')'D(E - \hat{y}a'). \tag{42}$$

The single fit σ_2 is the second term on the right. This component equals zero in case of multiple variables only. The multiple fit is, given that $X'MX = I$ and $e'Mx = 0$,

$$\sigma_1 = \text{tr } (X - \Delta E)'M(X - \Delta E), \tag{43}$$

$$= p - \text{tr } E'DE. \tag{44}$$

Single fit. The single fit is, cf. (42),

$$\sigma_2 = \text{tr } (E - \hat{y}a')'D(E - \hat{y}a'), \tag{45}$$

$$= \text{tr}(E - \bar{y}a')'D(E - \bar{y}a') + (\bar{y} - \hat{y})'D(\bar{y} - \hat{y}). \tag{46}$$

The fit of the direction cosines σ_{2a} is, given that $a'a = 1$,

$$\sigma_{2a} = \text{tr}(E - \bar{y}a')'D(E - \bar{y}a'), \tag{47}$$

$$= \text{tr } E'DE - \bar{y}'D\bar{y} . \tag{48}$$

The fit of the optimally scaled vectors σ_{2b}, given that $\hat{y}$ minimizes σ_{2b} with respect to $\hat{y} \in K$ and that

$$\hat{y}'D\bar{y} = \hat{y}'D\hat{y}, \tag{49}$$

is

$$\sigma_{2b} = \text{tr } (\bar{y} - \hat{y})'D(\bar{y} - \hat{y}), \tag{50}$$

$$= - \hat{y}'D\hat{y} + \bar{y}'D\bar{y}. \tag{51}$$

Thus the overall stress equals

$$\sigma = p - \hat{y}'D\hat{y}. \tag{52}$$

The most right term is called the overall fit.

Define u^k as the parameter set of the k-th iteration,

Φ as the operation of the three step algorithm,

σ as the loss, bounded from below.

Suppose the algorithm generates an infinite sequence u^k, none of the u^k is a stationary point,then

$$\sigma(u^k) > \sigma(u^{k+1}) > 0 \ , \tag{53}$$

$$\Phi(u^k) = u^{k+1} \ , \tag{54}$$

$$u^k \text{ belongs to the compact set } u \ . \tag{55}$$

From (48) it follows that $\sigma(u^k)$ converges to σ^*.

Bolzano-Weierstrass shows that u^k has an accumulation point u^*, i.e. there is a subsequence (u^ℓ) such that $u^\ell \to u^*$. Define $(u^{\ell+1})$. this sequence again has a subsequence (u^{v+1}) converging to, say, u^{**}. And finally we construct (u^v) which is a subsequence of (u^ℓ) and which converges consequently to u^*. For example

if (u^ℓ) is $u^{(1)},u^{(3)},u^{(5)},u^{(7)},u^{(9)},\ldots$

then $(u^{\ell+1})$ is $u^{(2)},u^{(4)},u^{(6)},u^{(8)},u^{(10)},\ldots$

if (u^{v+1}) is $u^{(2)},u^{(6)},u^{(10)},\ldots$

then (u^v) is $u^{(1)},u^{(5)},u^{(9)},\ldots$

Because Φ is continuous and $u^{v+1} = \Phi\ (u^v)$ it follows that $u^{**} = \Phi\ (u^*)$.

Because σ is continuous and $\sigma(u^k) \to \sigma^*$ it follows that $\sigma(u^{**}) = \sigma(u^*)$.

But, if u^* is not a stationary point then $\sigma(u^{**}) > \sigma(u^*)$, because of $u^{**} = \Phi(u^*)$.

Thus u^* is stationary. We have proved that all accumulation points of u^k are stationary points with the same function value σ^*.

THE PRINCALS PROGRAM DESCRIPTION

PRINCALS is also the name of a computer program written in ANSI FORTRAN IV. The program consists of two phases which both use the same iterative algorithm and code. The size of the code is 2800 statements.
The first phase is the iterative computation of an SVD of the raw data; the second phase is the iterative computation of the final generalized PCA with mixed measurement levels with, except for multiple variables, the results of the first phase as initial configuration. Both phases use the same algorithm and code but with different options. To compute the initial SVD the program uses the numerical measurement level only, to compute the final solution all measurement levels are available. The configuration used to start the computation of the SVD is a normalized random configuration. In both phases there are mutually independent and extensive output facilities.

	PHASE I	PHASE II
the initial configuration	random	SVD
the measurement levels	single numerical	all
I/O options	all	all
the model	single metric PCA	metric or non-metric, single or multiple PCA

Table 1

The algorithm. We minimize alternatingly over X, Y, and A. We start with X^0 and A^0, where the superscript '^{0}' stands for the initial or preceding values and the superscript '$^+$' indicates the updates.

The following steps are alternated:

$$(1) \qquad E_j^0 = D^{-1}\Delta_j M_j X^0$$

$$(2) \qquad \hat{y}_j^0 \underset{\text{proj. } K}{=} E_j^0 a_j^0$$

$$(3) \qquad a_j^{\dagger} = \hat{a}_j(\hat{a}_j'\hat{a}_j)^{-\frac{1}{2}}$$

$$(4) \qquad X^{\dagger} = M^{\frac{1}{2}}\breve{X}(\breve{X}'M\breve{X})^{-\frac{1}{2}}$$

where

$$\hat{a}_j = \hat{y}_j^{0\prime}E_j^0 \ ,$$

and

$$\breve{X} = M^{-\frac{1}{2}} \Sigma_{j=1}^m \Delta_j \hat{y}_j^0 a_j^{\dagger} \ .$$

The steps (1), (2) and (3) are executed subsequently per variable. To obtain more precise updates for a_j and $\hat{y}_j$ the steps (2) and (3) are computed alternatingly till the updates are stable. In most cases one inner iteration is sufficient as is to be expected according to theory. In case of only multiple variables, only step (1) and step (4) are computed alternatingly. The algorithm is relaxed by computing step (4) every three iterations. We have not yet decided upon the optimal number of such non-orthogonalizing iterations.

After the final configuration is stable enough to meet the convergence criterion, the solution is rotated to its principal components and the corresponding eigenvalues are computed. All other quantifications are recomputed with the rotated solution.

An extra output facility are the plots of the centroids of the points of the observations in the same category. These plots are available per variable. If compared with the single category scores of a variable, which are in the same plot, they can serve as indications for the way the observations are spread on the hyperplanes. They are called multiple category scores.

EXAMPLES

We will illustrate the use of PRINCALS with some examples. The analysis of the Dutch Parliament data (Daalder & Rusk,1972, De Leeuw, 1973, and Daalder & Van de Geer, 1977) is shown extensively with all computer output. The remaining examples are reported more concisely.

The Dutch Parliament data. These data consist of 141 preference orders of 12 dutch political parties. These preference orders stem from 141 members of the dutch parliament in 1968.

Name of the party	Party -labels other plots	Party -labels Boot-straps	Description	Number of respondent members	Labels for the members of parliament
CPN	CPN	1	communists extreme left	refused to answer	-
PSP	PSP	2	pacifists intelligent left	4	P
PvdA	PVA	3	socialists constructive left	37	L
D'66	D66	4	democrats intelligent left	7	6
PPR	PPR	5	radicals intelligent left	refused to answer	-
KVP	KVP	6	catholics middle	42	K
ARP	ARP	7	protestant middle	15	A
CHU	CHU	8	protestant middle	12	U
VVD	VVD	9	liberals constructive right	17	V
BP	BOP	10	farmers extreme right	4	B
SGP	SGP	11	dissident calvinist extreme right	2	S
GPV	GPV	12	dissident calvinist extreme right	1	G

Table 2 Description of data and codes.

The model is that each preference order is represented as a vector on which the projections of the party-points are situated in that order of preference.

We want to find 12 party points in the solution space, whose projections on these preference vectors are monotone with the observed preference orders, i.e. this is the classical application of the vector model on preference data cf. MDPREF (Carrol ,1972), also known as a decomposition technique (Heiser & De Leeuw, 1979 a). We proceeded by analyzing the 12 x 141 (sic.) matrix of preference orders as ordinal data in four dimensions. The number four is rather arbitrary but we have to draw more than two dimensions to illustrate the relatively bad fit of some preference orders to the first two dimensions.. If we compute only two dimensions all preference vectors would be of equal length as in figure 4.

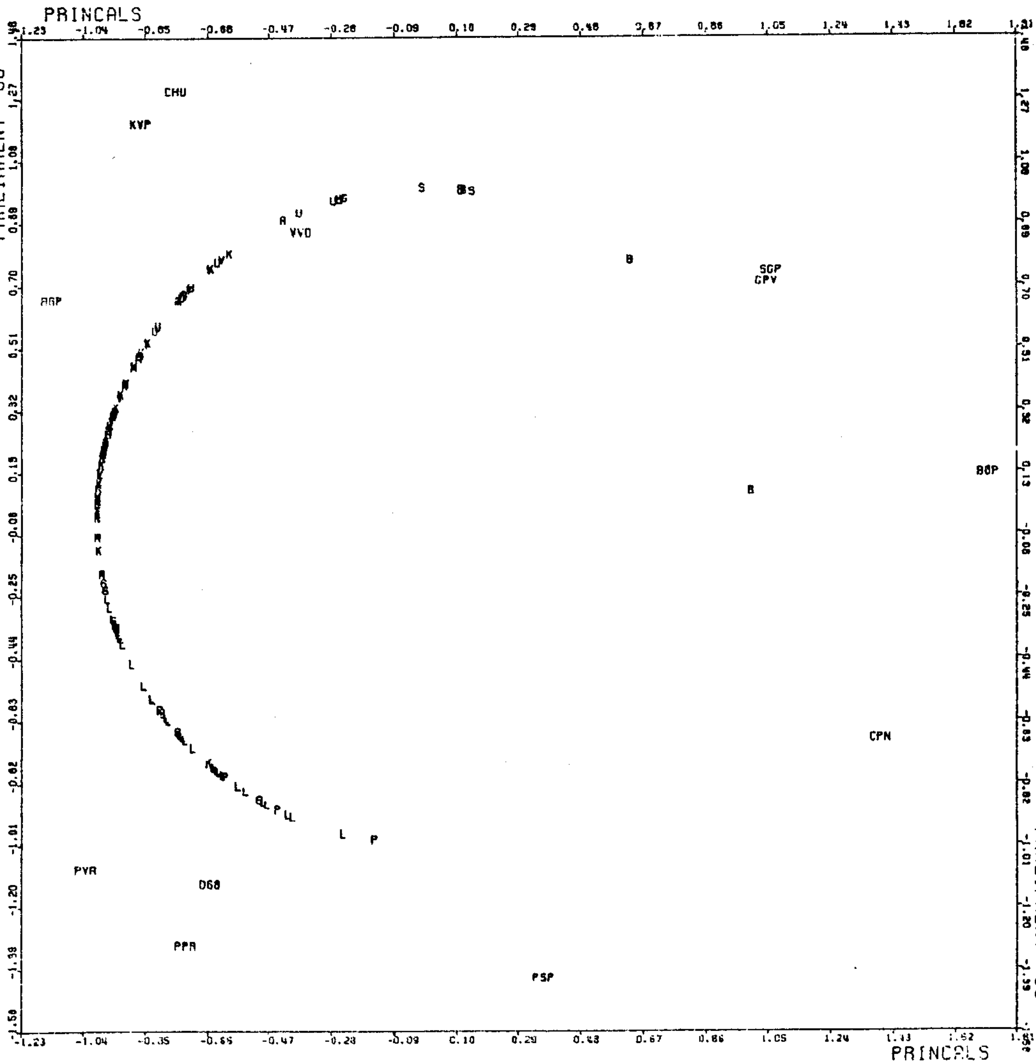


FIGURE 4 PRINCALS; ORDINAL; DUTCH PARLIAMENT 1968

The first two dimensions of a two dimensional analysis

In the two-dimensional case, the normalization,

$$\Sigma_{s=1}^{p} a'_{js} a_{js} = 1,$$

creates a circle on which the preference vectors, or direction cosines, have to be situated. For p = 3 this would be a sphere. The projections of points on such a hyper-sphere onto the first two axes are not necessarily on a circle. The length of these projections illustrates the relative fit of those preference orders in the first two dimensions. If they are on a circle this means that fit perfectly in two dimensions, provided of course that **it is** a p > 2 analysis. Figure 5 shows that for instance the VVD members are not fitting so nicely as the others.

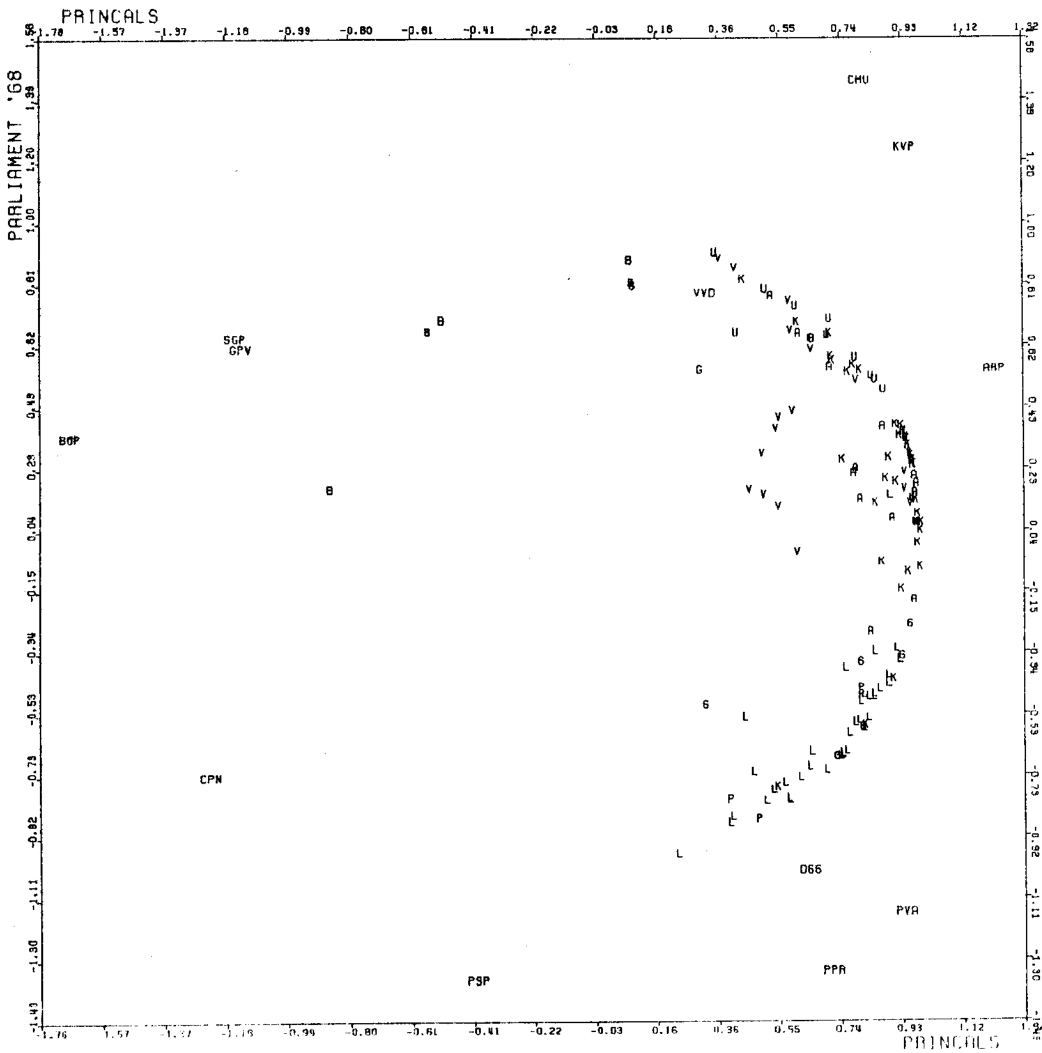


FIGURE 5 PRINCALS; ORDINAL; DUTCH PARLIAMENT 1968

The first two dimensions of a four dimensional analysis

Both analyses have the same first two eigenvalues and the party-configurations are also equal. There is only a difference in reflection.
The numerical solution in figure 6 is hardly different from the **ordinal** solution in figure 5. The ordinal 'amelioration' of the numerical configuration did not bring much. Only the eigenvalues increased somewhat from .56 and .25 for the numerical analysis to .64 and .28 for the ordinal solution, but the basic party configuration remained the same.. This underlines the **point of** view that ordinal analysis is not automatically superior to metric analysis if random error is present (Woodward & Overall, 1976).

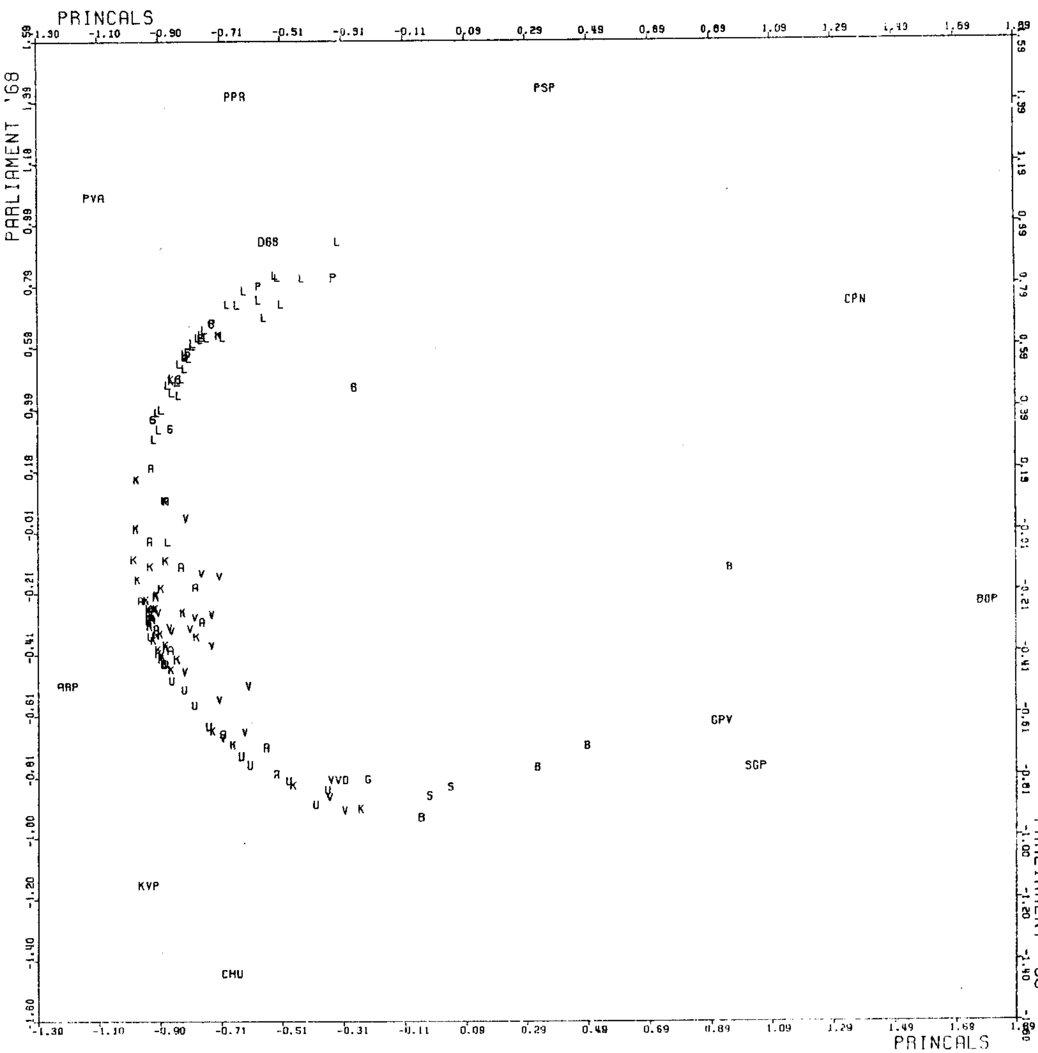


FIGURE 6 PRINCALS; NUMERICAL; DUTCH PARLIAMENT 1968

The first two dimensions of a four dimendional analysis

In order to have a closer look at the stability of the ordinal solution we applied a bootstrap technique of Efron (1979). The bootstrap is a generalization of the Quenouille-Tukey jackknife non-parametric method for estimating the bias and variance of a statistic. We draw 10 ' bootstrap' samples of the size 141 from our dataset *with replacement*. We analyse these samples with PRINCALS and plot the party-points together in one plot, see figure 7 . The different samples are identified with letters A - K , I, excluded. The points marked X indicate the original configuration. The labels for the parties are as in table 1.The eigenvalues range from .61 to .69 for the first axis and from .23 to .30 for the second axis. The 95% confidence interval for the first eigenvalue is .64 ± .05 and for the second eigenvalue it is .28 ± .04 . All points seem to be nicely grouped around the original party points. Only the communists party uses a somewhat bigger area in the plot. This is caused

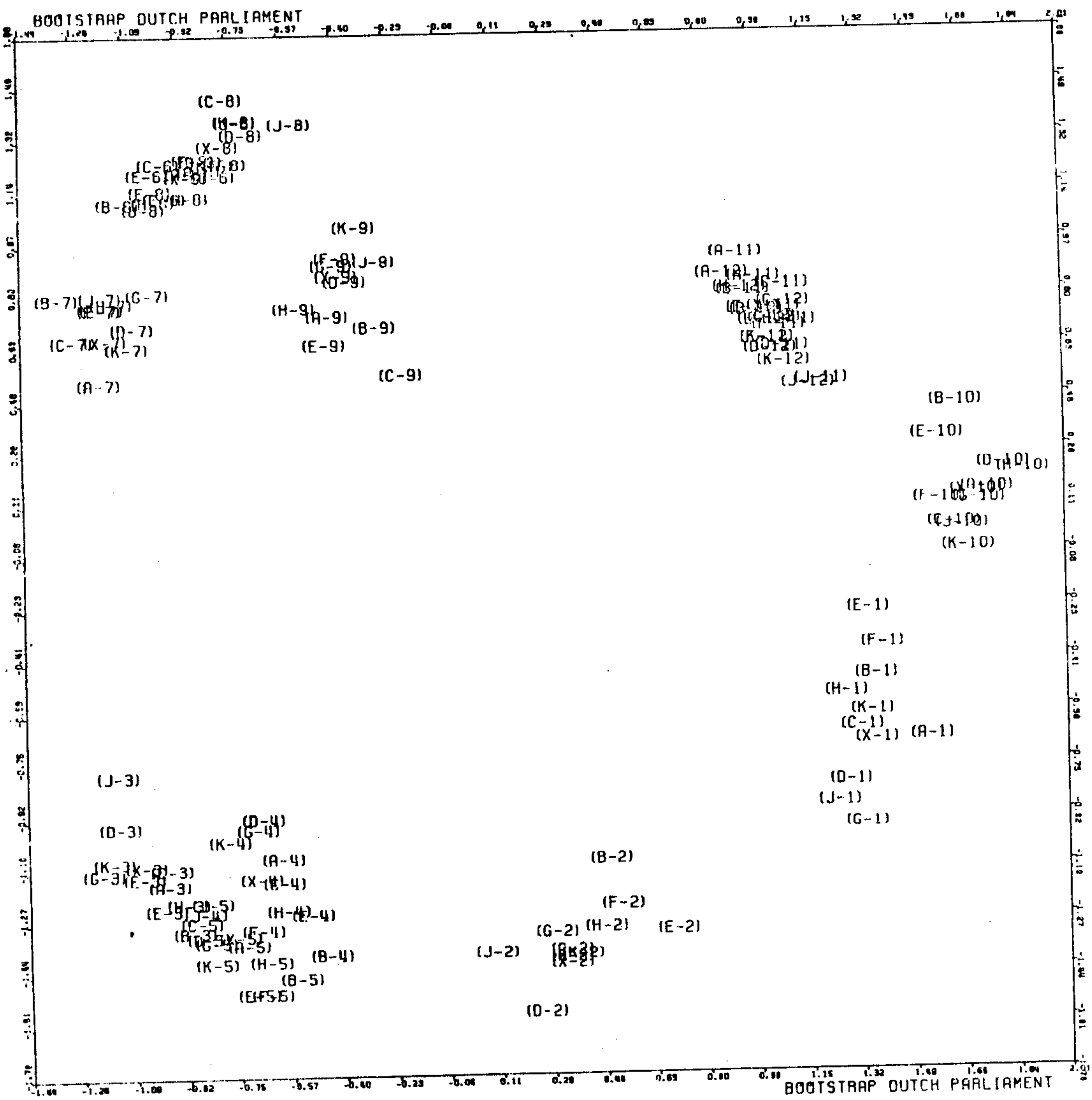


FIGURE 7

by the fact that this party is the least preferred by a great number of respondents, belonging to a variety of other parties.

Preferences for progeny composition

The data consist of preference rank numbers of 86 psychology students at the Leuven university given to 16 types of progeny composition. Extensive analyses of these data are reported by Heiser & De Leeuw (1979b) and by Delbeke (1978). The 16 possible progeny compositions are all combinations from no offspring (=0,0) to three sons and three daughters (=3,3). The first digit in the code indicates the number of sons and the second digit the number of daughters. We are able to use these data by courtesy of Mr. Delbeke.

The data are analysed as ordinal data, collected in a 16 x 86 (sic) matrix of preference orders. Each respondent is represented by a direction cosine on which the projections of the progeny composition points are ordered according to the respondent's preference. .

Ordinal unfolding of these data is the most obvious way of analyzing but it is practically not very satisfactory although stress is low (of course) as is shown by Delbeke (1978). Heiser & De Leeuw find a nice and stable configuration shaped like a grid of progeny composition points. They applied SMACOF III, a sophisticated metric unfolding technique. Although in their analysis the most unpopular progeny type is an unstable point on the outside of the grid. Metric analysis may seem not so approppriate but it is apparently more useful for this data set. This point will be illustrated in the sequel.

The upper left hand configuration in figure 8 is a singular value decomposition (SVD) of the raw data matrix; it serves as the initial metric configuration in all our ordinal PRINCALS analyses of these data and it is supplied by the PRINCALS program. The majority of the direction cosines is in a fan shaped sheaf pointing downwards to the lower right hand corner of the plot.

The general preference direction in this plot is from the least preferred item, 0.0, to the most popular item, 2.1. There is a sex bias in favor of boys and the idealized number of children is two or three.

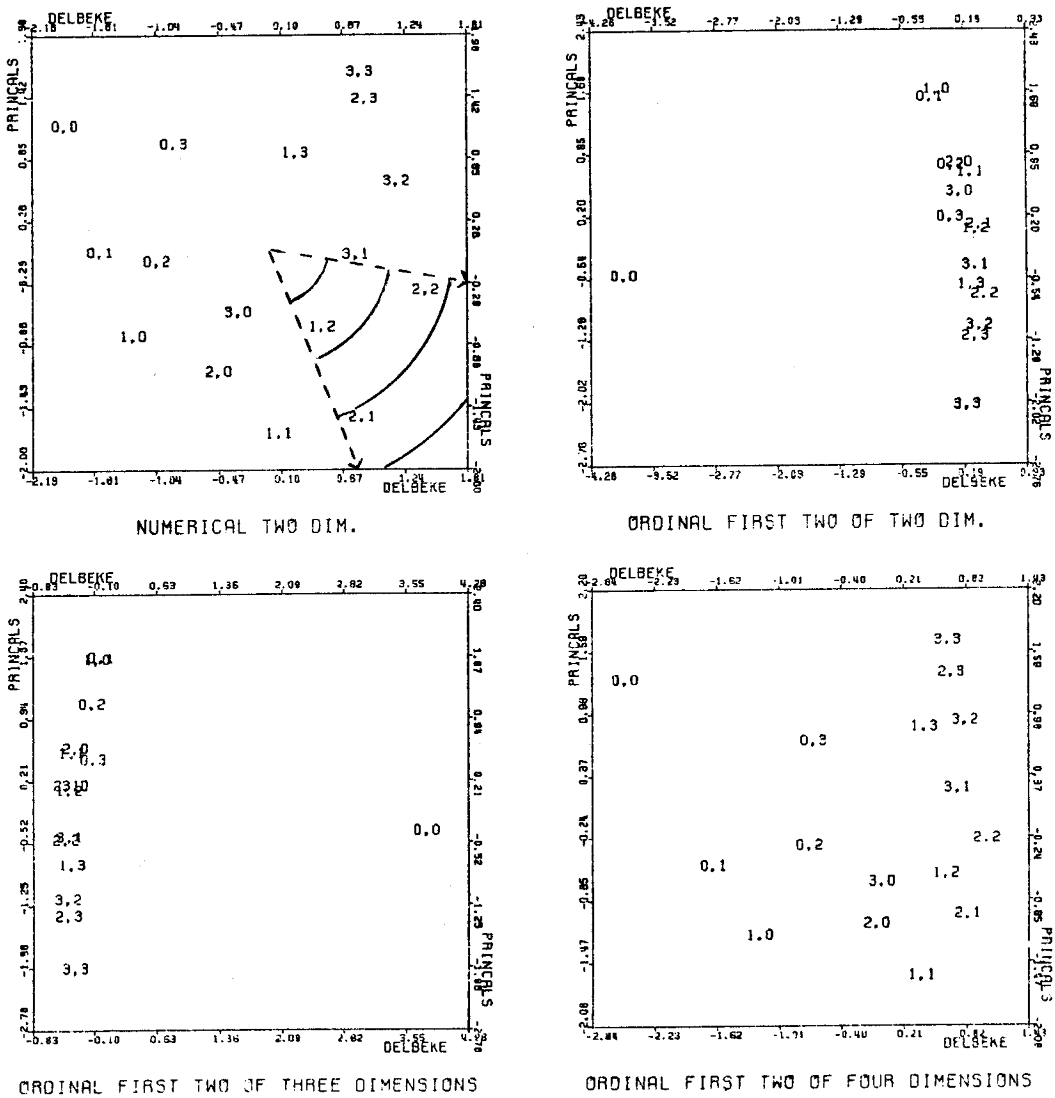


Figure 8

The upper right-hand configuration shows that in a two-dimensional ordinal analysis the configuration degenerates into a dominant (and trivial, compared with the results of Heiser et al. and compared with the initial configuration) difference between having children and having no children at all on the first axis. The progeny points are ordered on the second axis according to the number of children. The idealized number of children and the sex bias are not recognized in this plot. The first two dimensions of the three-dimensional ordinal analysis gives the same result. Only the four-dimensional ordinal analysis has a less degenerate structure in which the more unpopular combinations are positioned in such a way that their projections are on the opposite end of the preference direction of the metric solution. The overall configuration resembles the metric solution very much.

We used the same convergence criterion for these four PRINCALS analyses. This entails that the high dimensional solutions are less accurate than the low dimensional ones. In the ordinal case the algorithm needs more iterations before the stress stabilizes as the number of dimensions increases. See also table 3 . The iterative process stops comparatively to early and the solution is only slightly different from the initial metric configuration. The ordinal approach in this case is so to speak not able to " get out " of the initial configuration.

Dimensions	→	1	2	3	4	5	6	number of iterations
	↓ 1	.89						20
	2	.85	.97					27
ordinal	3	.84	.95	.99				22
	4	.66	.86	.94	.99			16
	5	.62	.83	.92	.96	.99		8
	6	.60	.82	.91	.95	.98	.99	5
metric	6	.57	.79	.88	.91	.94	.95	5

Table 3

The cumulative eigenvalues and the numbers of iterations for six PRINCALS runs with a fixed convergence criterion.

For this example we use PRINCALS as a one-dimensional technique to transform the data to obtain a data reduction, or a one-dimensional scale. Both interpretations amount to maximizing the homogeneity of the data and a measure for this maximal homogeneity is the first eigenvalue. We analyze a random sample of 1825 respondents from the generation that left primary school in 1965 in The Netherlands. The observations are scored on 25 background variables. See table 4. The data are collected and extensively described by Kropman & Collaris (1974) and by Collaris & Kropman (1978). Analyses of these data are reported by Dessens & Janssen (1979), Dronkers (1978), Dronkers & Jungbluth (1979) and De Leeuw & Stoop (1979). These variables can roughly be divided into four types: binary variables, variables of the rating scale type, pure categorical variables and discretized variables. Because the description of all the categories would be too extensive we confine ourselves to state the meaning and type of each variable in table 4 . The PRINCALS fit and the HOMALS discrimination measures are also in table 4 . These quantifications are the squared correlations respectively the correlations of the variables with the one-dimensional scale and also the variable-components of the homogeneity.

MEANING	CODE	CAT	TYPE	PRINCALS FIT			HOMALS DISC
				NUM	ORD	NOM	MEAS.
The parental approval of the child's choice of secundary education in 1970.	INS	2	bin	.005	.003	.003	.10
The relative importance for girls of the choice of the future profession	BMB	4	sca	.191	.179	.172	.41
The degree of urbanization of the place of residence in 1965	URB	4	cat	.028	.031	.032	.18
The occupational level of the father in 1965.	BVA	7	cat	.318	.393	.391	.63
The first chosen secondary education after primary school in 1965	TON	5	cat	.741	.797	.799	.89

compared with the results of Heiser et al. and compared with the initial configuration) difference between having children and having no children at all on the first axis. The progeny points are ordered on the second axis according to the number of children. The idealized number of children and the sex bias are not recognized in this plot. The first two dimensions of the three-dimensional ordinal analysis gives the same result. Only the **four-dimensional** ordinal analysis has a less degenerate structure in which the more unpopular combinations are positioned in such a way that their projections are on the opposite end of the preference direction of the metric solution. The overall configuration resembles the metric solution very much.

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The first chosen secondary education after primary school in 1965	TON	5	cat	.741	.797	.799	.89

table continued

MEANING	CODE	CAT	TYPE	PRINCALS FIT NUM	ORD	NOM	HOMALS DISC MEAS.
The score on aptitude test for higher education.	PRE	9	dis	.642	.650	.647	.80
The score on a test for the choice of profession on the scale L, especially computed for those who want a lower vocational training .	BIL	9	dis	.042	.063	.059	.24
The same as BIL but scored on the scale M, especially computed for those who want a higher educational training.	BIM	9	dis	.160	.192	.188	.43
The teacher's advise for the optimal choice of the secondary education.	ADV	4	cat	.591	.654	.657	.81
As PRE but the average score of the whole class.	KGS	4	dis	.426	.429	.425	.65
The number of pupils in the last form in 1965.	LL6	4	dis	.003	.010	.015	.12
Parental interest in the achievements of their child on school according to the teacher.	INT	2	bin	.150	.123	.123	.35
The level of parental aspiration for their child's professional career.	ASO	6	sca	.036	.050	.048	.22
The level of aspiration of the pupil in respect to its own professional career in 1970.	ASL	6	sca	.021	.063	.071	.26
The parental sensitivity to the opinion of others about the choice of the secondary education of their child.	OOA	5	sca	.051	.052	.052	.22
The amount of force the parents apply to carry their choice of secondary education through.	DWO	5	sca	.008	.009	.008	.09

table continued

MEANING	CODE	CAT	TYPE	PRINCALS FIT NUM	PRINCALS FIT ORD	PRINCALS FIT NOM	HOMALS DISC. MEAS.
Whether the pupil has been on a nursery school.	KLS	2	bin	.020	.023	.023	.15
Whether the pupil had to stay in a class for an extra year.	DLO	2	bin	.177	.150	.145	.38
The educational level of the father in 1970.	OPV	7	cat	.391	.424	.419	.65
The educational level of the mother in 1970.	OPM	7	cat	.238	.265	.263	.51
The number of children in the family.	AKG	9	dis	.052	.056	.058	.24
Whether there are different types of education available at the secondary school.	AOS	2	bin	.176	.201	.212	.46
The number of pupils at the secondary school.	LLS	8	dis	.103	.124	.139	.37
The number of extra curricular activities available at the secondary school.	EXT	6	dis	.084	.176	.353	.60
The highest educational level reached after leaving secondary school.	EIN	12	dis	.712	.735	.740	.86
Total homogeneity				.214	.234	.242	.242

Table 4

The discretized and rating scale variables have ordered categories and the categories of the so-called typical categorical variables are chosen on such a way that that they are ordered in a conceptual sense, i.e. from lower to higher educational level etc.. For a more elaborate description of the variables and their categories see Dronkers (1979). The data were collected in 1965,1970 and in 1974.

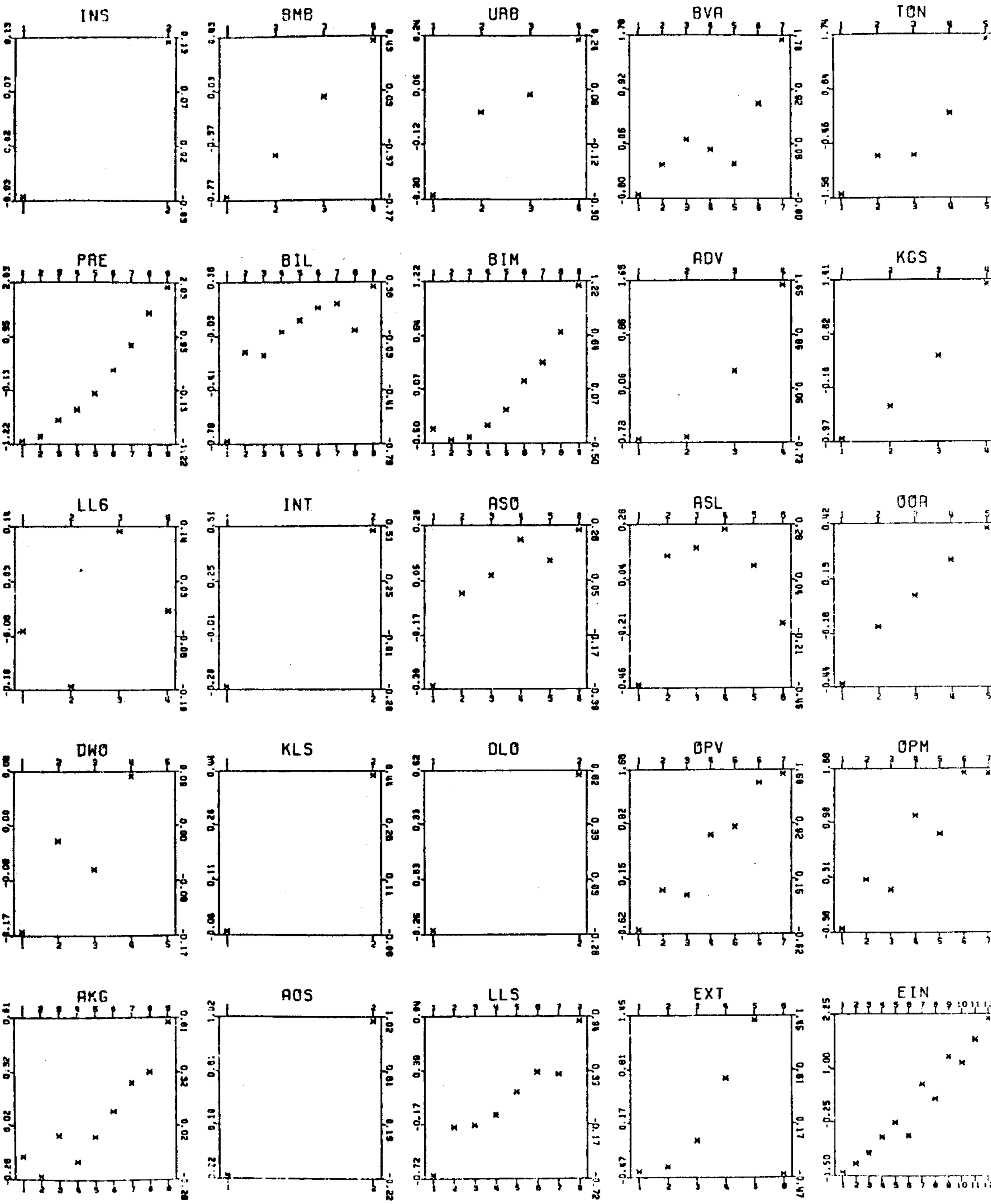

Figure 9

One-dimensional non-linear single nominal transformations of 25 variables

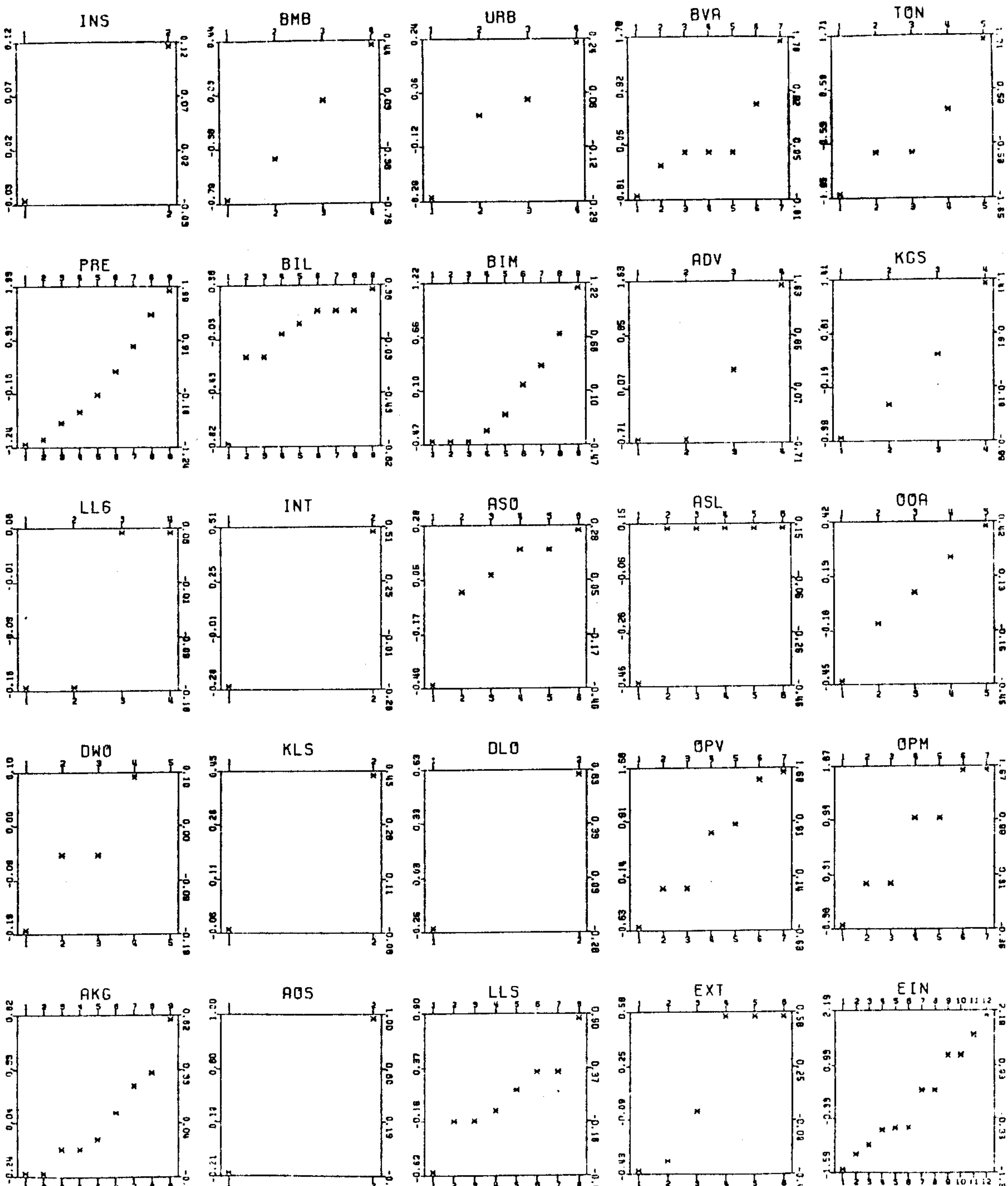

Figure 10

One-dimensional linear single ordinal transformations of 25 variables

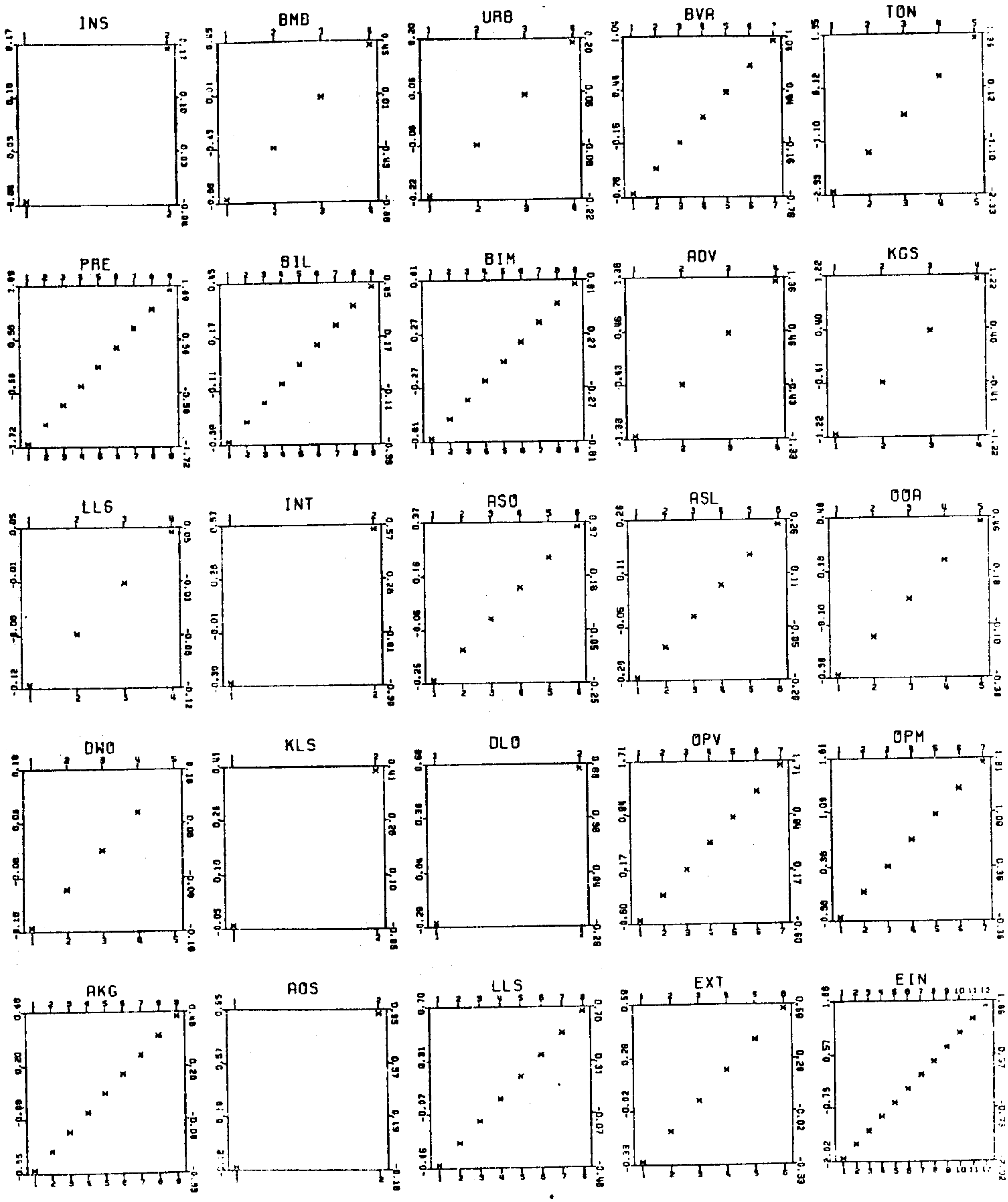

Figure 11

One-dimensional linear single numerical transformations of 25 variables

The tendency of the one-dimensional, and therefore non-linear, single nominal transformations of PRINCALS is, like in HOMALS, to transform to multinormality and to correct for the skewness of the marginals (De Leeuw & Stoop,1979). Our non-linear transformations turn out to be almost linear. See figure 9 . This is illustrated by the relative small gain in homogeneity of the non-linear transformations compared with the linear approach, and even this difference in homogeneity is mainly caused by a coding error in the variable EXT, where a category, coded 6, had to be coded 0.

The transformations are plotted against the category numbers in figure 9 to 11 to make a visual inspection possible. The relative weight of the numerical transformations is indicated by the fit in table 4.

A global assumption of linearity and multinormality seems to be justified, especially after some recoding. The one-dimensional PRINCALS transformations may thus be used as input for classical canonical analysis.

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P R I N C A L S              A PROGRAM FOR PRINCIPAL COMPONENTS ANALYSIS         JAN VAN RIJCKEVORSEL & JAN DE LEEUW
                           OF DISCRETE DATA WITH MIXED MEASUREMENT LEVELS
VERSION    3.02                                                                      DEPARTEMENT OF DATATHEORY

 APRIL   1979                                                                        THE UNIVERSITY OF LEIDEN

                                                                                       WASSENAARSEWEG 80

                                                                                            LEIDEN

                                                                                       THE NETHERLANDS

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|JOB NR. 1|
 =========

INPUT-DATA SPECIFICATIONS:
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NAME -     PRINCALS WITH THE PARL68 DATA  ORDINAL EN NUMERICAL  RECODED


                         = PROBLEM PARAMETERS =


NOBS - NUMBER OF OBSERVATIONS                                                 12
NVR1 - TOTAL NUMBER OF VARIABLES IN THE DATAMATRIX                           141
NVR2 - NUMBER OF ANALYSIS-VARIABLES                                          141
NDIM - NUMBER OF DIMENSIONS                                                    4
MAXC - MAXIMUM NUMBER OF CATEGORIES OVER ALL VARIABLES                        13
ISUC - TOTAL NUMBER OF CATEGORIES OF THE                                    1833
       ANALYSIS-VARIABLES

                         = ANALYSIS PARAMETERS =


MAXI - MAXIMUM NUMBER OF ITERATIONS TO COMPUTE THE FINAL  CONFIGURATION       75
EPSI - CONVERGENCE CRITERION FOR THE FINAL CONFIGURATION               0.50E-04
NITIN- MAXIMUM NUMBER OF ITERATIONS TO COMPUTE THE INITIAL  CONFIGURATION     20
EPSIN- CONVERGENCE CRITERION FOR THE INITIAL CONFIGURATION             0.50E-03
```

```
                 = INPUT/OUTPUT PARAMETERS =

INPU - UNIT NUMBER OF THE DATAMATRIX                                          2

IDAT - PARAMETER INDICATING WHETHER OR NOT (1 AND Ø RESP.)                    1
       TO PRINT THE DATAMATRIX

IWR1 - PARAMETER INDICATING WHETHER OR NOT TO PRINT                          11
       OBSERVATION- AND CATEGORY SCORES
           =Ø: NO PRINT
           =1: OBSERVATION SCORES ONLY
           =2: BOTH
           =3: CATEGORY SCORES ONLY

IWR2 - PARAMETER INDICATING WHETHER OR NOT (1 AND Ø RESP.)                   11
       TO PRINT THE ITERATION HISTORY

IPLO - PARAMETER INDICATING WHETHER OR NOT AND HOW TO                        11
       PLOT OBSERVATION AND CATEGORY SCORES
           =Ø: NO PLOT
           =1: OBSERVATION SCORES, UNPARTITIONED, AND DIRECTION COSINES
           =2: PLOT PARTITIONED ACCORDING TO THE USERS REQUEST

IOUT1- UNIT-NUMBER FOR OUTPUT OF THE OBSERVATION SCORES TO                    Ø
       CARD, TAPE OR DISK (=Ø: NO OUTPUT OF THIS KIND

IOUT2- LIKEWISE FOR THE CATEGORY SCORES                                       Ø

IOUT3- LIKEWISE FOR THE SCALED DATA VECTORS                                   Ø

ILEV - MEASUREMENT LEVEL FOR ALL VARIABLES                                    3
           =Ø: MIXED MEASUREMENT LEVELS
           =1: ONLY MULTIPLE NOMINAL VARIABLES
           =2: ONLY SINGLE NOMINAL VARIABLES
           =3: ONLY SINGLE ORDINAL VARIABLES
           =4: ONLY SINGLE NUMERICAL VARIABLES

IPRIN- OUTPUT SUPPRESSING PARAMETER                                           1
           =Ø: NO OUTPUT OF THE INITIAL CONFIGURATION
           =1: IDENTICAL OUTPUT OPTIONS FOR BOTH THE INITIAL AND FINAL CONFIGURATION
           =2: OUTPUT OPTIONS FOR BOTH CONFIGURATIONS ARE FULLY SPECIFIED BY THE USER

ICAT- NUMBER OF CATEGORIES FOR ALL VARIABLES                                 13
           =ICAT :THE CATEGORY NUMBERS FOR ALL VARIABLES EQUAL ICAT
           =Ø: THE CATEGORY NUMBERS FOR ALL VARIABLES ARE DIFFERENT



THE NUMBER OF CATEGORIES FOR ALL VARIABLES IS:   13


FMT1 - FORMAT TO READ THE DATAMATRIX: (4(1X,3ØI2/),1X,21I2)


LISTING OF THE DATAMATRIX:
```

VARIABLES

OBSERVATIONS	1	2	3	4	5	6	7	8	9	1Ø	11	12	13	14	15	16	17	18	19	2Ø	21	22	23	24	25
1	4	4	11	7	1Ø	4	4	2	4	6	2	4	1	4	2	4	1	7	7	2	4	4	4	4	2
	4	4	4	2	4	6	2	1	2	4	4	8	3	4	4	4	4	2	2	4	6	4	4	2	3
	1	5	1	3	1	2	5	5	2	2	4	1	1	4	4	4	2	2	4	4	4	4	6	3	2
	1	1	3	2	1	4	1	2	1	4	2	4	4	1	1	2	4	2	4	4	2	1	1	1	3
	2	1	2	1	2	1	1	2	3	1	3	1	1	3	1	3	4	1	2	3	2	2	2	2	1
	2	3	2	1	2	1	1	1	4	1	1	4	6	1	1	1									
2	12	12	12	12	8	1Ø	7	9	8	1Ø	8	9	2	11	7	1Ø	7	8	9	5	1Ø	8	8	5	8
	8	1Ø	4	5	1Ø	9	8	9	1Ø	8	9	9	9	8	9	7	5	8	6	11	6	8	8	5	8
	5	6	2	3	4	8	2	6	3	3	7	3	5	5	5	8	5	3	3	7	8	4	2	3	3
	7	5	5	4	4	5	5	4	5	4	3	4	4	3	3	6	4	3	6	4	6	5	5	3	5
	3	2	4	2	3	2	5	6	2	3	3	4	3	4	3	3	3	3	3	5	4	3	5	1	3
	3	2	3	5	1	2	4	5	3	2	3	4	6	2	2	2									
3	11	11	1Ø	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	9	1Ø	8	6	1Ø	1Ø	8	11
	8	1Ø	1Ø	9	8	11	9	1Ø	8	11	11	7	6	9	6	6	9	7	9	11	11	8	5	9	9
	1Ø	8	11	8	9	7	11	9	6	8	7	8	9	1Ø	8	9	8	8	1Ø	8	9	8	1Ø	1Ø	11
	6	5	8	8	7	5	9	4	9	8	8	5	5	7	9	3	8	1Ø	8	8	1Ø	11	1Ø	3	4
	7	9	8	1Ø	9	7	9	1Ø	7	5	5	4	6	5	5	4									
4	8	9	9	9	6	11	8	1Ø	1Ø	8	9	8	8	9	9	9	1Ø	1Ø	1Ø	1Ø	9	1Ø	9	11	1Ø
	9	9	1Ø	1Ø	9	1Ø	1Ø	1Ø	9	11	1Ø	7	1Ø	9	1Ø	1Ø	12	12	12	12	12	12	12	7	1Ø
	8	4	6	7	6	1Ø	4	7	4	4	9	6	7	8	7	7	5	6	2	9	1Ø	7	3	8	7
	6	6	9	6	6	6	6	8	7	9	9	7	1Ø	7	6	5	1Ø	6	7	4	6	9	6	6	7
	5	6	5	6	8	6	7	6	6	7	7	6	4	5	4	6	7	9	5	7	11	7	6	7	5
	8	6	11	11	1Ø	8	11	11	1Ø	4	2	4	6	4	4	6									
5	1Ø	1Ø	8	1Ø	11	9	11	11	11	11	11	11	9	1Ø	11	11	11	1Ø	11	11	11	11	11	1Ø	11
	1Ø	11	11	11	11	11	11	11	11	1Ø	11	1Ø	11	1Ø	11	11	11	11	11	9	6	11	11	6	12
	6	8	11	8	7	5	3	1	9	8	12	9	9	7	9	5	5	8	1	12	12	8	1	11	6
	5	11	1Ø	9	8	9	8	11	8	4	6	6	5	9	4	7	9	9	11	1Ø	6	7	9	8	8
	7	4	9	4	9	3	6	4	7	6	6	3	6	6	7	3	2	2	4	6	3	6	7	4	2
	6	7	6	4	6	3	7	8	6	3	4	4	6	3	3	5									
6	6	7	5	5	5	7	6	5	7	4	6	5	6	7	8	4	6	4	4	7	6	6	7	8	5
	4	7	4	7	4	4	6	7	6	7	5	3	7	5	7	8	8	6	8	7	6	7	7	12	3
	12	12	12	12	12	12	12	12	12	12	8	12	12	12	12	12	12	12	12	8	7	12	12	12	12
	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	1Ø	4	1Ø	8	11	1Ø	1Ø	8	11	9
	1Ø	1Ø	1Ø	1Ø	1Ø	8	1Ø	11	1Ø	1Ø	1Ø	7	9	1Ø	1Ø	1Ø	1Ø	8	1Ø	1Ø	9	9	9	11	1Ø
	11	1Ø	9	9	8	9	8	9	9	9	6	4	6	6	7	8									
7	9	8	7	8	9	8	1Ø	8	9	9	1Ø	1Ø	1Ø	8	1Ø	4	9	4	8	9	8	9	1Ø	9	9
	11	8	4	9	8	7	9	8	8	9	8	11	8	11	8	9	1Ø	1Ø	8	1Ø	6	9	9	11	9
	11	11	8	11	11	7	1Ø	11	11	1Ø	1Ø	11	11	11	11	11	11	11	11	1Ø	9	11	11	1Ø	11
	11	1Ø	8	11	11	11	1Ø	1Ø	1Ø	11	11	11	11	11	11	12	12	12	12	12	12	12	12	12	12
	12	12	12	12	12	11	11	9	11	11	11	1Ø	11	11	11	9	11	7	9	9	7	8	8	9	9
	9	4	7	7	7	1Ø	6	6	8	8	7	4	6	9	9	9									
8	7	5	4	6	4	5	5	5	5	5	7	6	7	6	6	4	5	4	5	6	5	5	5	6	7
	4	6	4	8	4	5	7	6	7	5	6	2	6	7	6	4	6	5	6	3	6	5	5	1Ø	6
	1Ø	9	9	1Ø	1Ø	6	11	8	1Ø	9	5	1Ø	1Ø	1Ø	1Ø	1Ø	1Ø	1Ø	1Ø	5	6	1Ø	1Ø	7	1Ø
	9	7	7	1Ø	1Ø	1Ø	9	7	11	1Ø	1Ø	1Ø	7	8	1Ø	11	11	11	9	9	11	11	11	9	1Ø
	11	11	11	11	11	12	12	12	12	12	12	12	12	12	12	12	12	11	11	11	8	1Ø	11	1Ø	11
	1Ø	11	1Ø	8	11	11	1Ø	7	11	1Ø	9	11	6	1Ø	1Ø	1Ø									
9	5	6	6	4	7	6	9	5	6	7	5	7	11	5	5	8	8	4	6	8	7	7	6	7	6
	1	5	4	6	4	8	5	5	5	6	7	6	5	6	5	4	7	7	1Ø	6	6	6	6	9	7
	8	7	7	6	9	9	8	9	7	7	6	8	8	6	8	9	8	9	8	6	5	6	8	6	8
	8	9	6	8	7	8	7	6	9	7	8	9	8	4	7	8	4	7	5	4	6	6	3	7	4
	7	7	3	7	6	7	8	1Ø	8	9	9	11	1Ø	8	8	11	9	12	12	12	12	12	12	12	12
	12	12	12	12	12	12	12	12	12	11	8	1Ø	6	8	8	7									
1Ø	1	1	1	1	1	1	1	2	1	1	1	1	3	1	1	4	2	4	1	1	1	2	1	1	1

	*	4	1	4	1	4	1	1	2	1	1	1	1	1	1	1	4	1	1	2	1	6	1	1	1	3
	*	2	1	3	3	2	1	1	2	1	1	1	2	2	3	1	1	1	1	5	1	1	1	4	3	1
	*	2	4	1	1	3	1	2	1	2	1	1	1	4	2	2	1	4	1	1	4	2	2	2	2	1
	*	4	3	1	3	1	4	2	2	1	2	3	2	2	2	2	3	1	4	1	1	1	1	1	8	6
	*	1	1	1	6	3	5	3	2	1	12	12	12	12	7	6	3									
11	*	2	2	2	2	3	3	2	5	2	2	4	2	4	3	3	4	3	4	2	3	2	2	2	2	3
	*	4	2	4	3	4	3	3	3	3	2	2	4	2	2	2	4	3	3	2	2	6	3	2	3	3
	*	4	3	5	3	5	3	7	4	6	6	2	4	3	2	3	3	5	4	7	2	3	4	9	3	4
	*	4	3	2	3	2	3	3	3	3	4	4	4	4	5	9	4	4	4	2	4	6	3	4	4	6
	*	8	8	6	5	4	9	4	6	5	5	3	9	8	9	6	8	5	5	7	2	5	4	3	6	7
	*	4	8	4	3	4	4	2	3	2	6	11	8	6	12	12	11									
12	*	3	3	3	3	2	2	3	5	3	3	3	3	5	2	4	4	4	4	3	4	3	2	3	3	4
	*	4	3	4	4	4	2	4	4	4	3	3	5	4	3	3	4	2	4	2	5	6	2	3	4	3
	*	4	2	4	3	3	2	6	3	5	5	3	5	4	1	2	2	5	5	6	3	2	4	7	3	5
	*	3	2	4	5	5	2	4	5	4	4	5	4	4	6	5	3	4	5	3	4	6	4	7	5	2
	*	9	9	7	9	5	1Ø	3	6	4	4	3	8	7	1	5	7	6	6	6	4	6	5	4	5	8
	*	5	5	5	2	5	6	5	4	5	7	1Ø	8	6	11	11	12									

ITYP - MEASUREMENT LEVEL FOR ALL VARIABLES IS: SINGLE ORDINAL

MARGINAL FREQUENCIES:

* CATEGORIES

VARIABLES	MISSING	1	2	3	4	5	6	7	8	9	1Ø
1	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
2	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
3	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
4	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
5	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
6	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
7	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
8	Ø	Ø	2	Ø	Ø	5	Ø	Ø	1	1	1
		1	1	Ø							
9	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
1Ø	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
11	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
12	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
13	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
14	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
15	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
16	Ø	Ø	Ø	Ø	7	Ø	Ø	Ø	1	1	1
		1	1	Ø							
17	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
18	Ø	Ø	Ø	Ø	7	Ø	Ø	1	1	Ø	2
		Ø	1	Ø							
19	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
2Ø	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
21	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
22	Ø	Ø	3	Ø	1	1	1	1	1	1	1
		1	1	Ø							
23	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
24	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							
25	Ø	1	1	1	1	1	1	1	1	1	1
		1	1	Ø							

26	*	Ø	1	Ø	Ø	6	Ø	Ø	Ø	1	1	1
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27	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
28	*	Ø	Ø	Ø	Ø	9	Ø	Ø	Ø	Ø	Ø	1
	*		1	1	Ø							
29	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
3Ø	*	Ø	Ø	Ø	Ø	7	Ø	Ø	Ø	1	1	1
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31	*	Ø	1	1	1	1	1	1	1	1	1	1
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32	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
33	*	Ø	1	1	1	1	1	1	1	1	1	1
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34	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
35	*	Ø	1	1	1	1	1	1	1	1	1	1
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36	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
37	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
38	*	Ø	1	1	1	1	1	1	1	1	1	1
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39	*	Ø	1	1	1	1	1	1	1	1	1	1
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4Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
41	*	Ø	Ø	Ø	Ø	6	Ø	Ø	1	1	1	1
	*		1	1	Ø							
42	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
43	*	Ø	1	1	1	1	1	1	1	1	1	1
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44	*	Ø	Ø	4	Ø	Ø	Ø	2	Ø	2	Ø	2
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45	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
46	*	Ø	Ø	Ø	Ø	Ø	Ø	11	Ø	Ø	Ø	Ø
	*		Ø	1	Ø							
47	*	Ø	1	1	1	1	1	1	1	1	1	1
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49	*	Ø	1	1	1	1	1	1.	1	1	1	1
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5Ø	*	Ø	Ø	Ø	5	Ø	Ø	1	1	1	1	1
	*		1	1	Ø							
51	*	Ø	1	1	Ø	2	1	1	Ø	3	Ø	1
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52	*	Ø	1	1	1	1	1	1	1	1	1	1
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53	*	Ø	1	1	1	1	1	1	1	1	1	1
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54	*	Ø	Ø	Ø	5	Ø	Ø	1	1	1	1	1
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55	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
56	*	Ø	1	2	1	Ø	1	1	1	1	1	1

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58	*	Ø	1	1	1	1	1	1	1	1	1	1
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59	*	Ø	1	1	1	1	1	1	1	1	1	1
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6Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
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66	*	Ø	1	1	1	1	1	1	1	1	1	1
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68	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
69	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
7Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
71	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
72	*	Ø	1	Ø	Ø	4	Ø	1	1	2	Ø	1
	*		1	1	Ø							
73	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
74	*	Ø	Ø	Ø	5	Ø	Ø	1	1	1	1	1
	*		1	1	Ø							
75	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
76	*	Ø	1	1	1	1	1	1	1	1	1	1
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77	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
78	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
79	*	Ø	1	1	1	1	1	1	Ø	2	1	1
	*		1	1	Ø							
8Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
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81	*	Ø	1	1	1	1	1	1	1	1	1	1
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82	*	Ø	1	1	1	1	1	1	1	1	1	1
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83	*	Ø	1	1	1	1	1	1	1	1	1	1
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84	*	Ø	1	1	1	1	1	1	1	1	1	1
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85	*	Ø	1	Ø	Ø	5	Ø	Ø	1	1	1	1
	*		1	1	Ø							
86	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							

87	*	Ø	1	Ø	Ø	4	Ø	1	1	1	1	1
	*		1	1	Ø							
88	*	Ø	Ø	Ø	Ø	5	1	Ø	1	1	1	1
	*		1	1	Ø							
89	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
9Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
91	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
92	*	Ø	Ø	Ø	Ø	7	Ø	Ø	Ø	1	1	1
	*		1	1	Ø							
93	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
94	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
95	*	Ø	Ø	Ø	Ø	7	Ø	Ø	Ø	1	1	1
	*		1	1	Ø							
96	*	Ø	Ø	2	Ø	Ø	Ø	6	Ø	Ø	1	1
	*		1	1	Ø							
97	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
98	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
99	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1ØØ	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø1	*	Ø	Ø	1	1	1	1	1	2	1	1	1
	*		1	1	Ø							
1Ø2	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø3	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø4	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø5	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø6	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø7	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
1Ø8	*	Ø	Ø	2	Ø	2	Ø	4	Ø	Ø	1	1
	*		1	1	Ø							
1Ø9	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
11Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
111	*	Ø	Ø	Ø	5	Ø	Ø	1	1	1	1	1
	*		1	1	Ø							
112	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
113	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
114	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
115	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
116	*	Ø	Ø	Ø	5	Ø	Ø	1	1	1	1	1
	*		1	1	Ø							
117	*	Ø	1	1	1	1	1	1	1	1	1	1

	*		1	1	Ø							
118	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
119	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
12Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
121	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
122	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
123	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
124	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
125	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
126	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
127	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
128	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
129	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
13Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
131	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
132	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
133	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
134	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
135	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
136	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
137	*	Ø	Ø	Ø	Ø	7	Ø	Ø	Ø	2	Ø	1
	*		1	1	Ø							
138	*	Ø	Ø	Ø	Ø	Ø	Ø	11	Ø	Ø	Ø	Ø
	*		Ø	1	Ø							
139	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
14Ø	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							
141	*	Ø	1	1	1	1	1	1	1	1	1	1
	*		1	1	Ø							

THE HISTORY OF ITERATIONS TO COMPUTE THE INITIAL METRIC CONFIGURATION

NUMBER OF ITERATIONS	TOTAL STRESS	MULTIPLE STRESS	SINGLE STRESS	AVERAGE NUMBER OF INNER ITERATIONS	DIFFERENCE BETWEEN THE LAST TWO ITERATIONS
1	Ø.Ø88912Ø	Ø.ØØØ2Ø26	Ø.Ø887Ø94	1.	Ø.Ø88912Ø
2	Ø.22Ø1954	Ø.ØØØ2Ø26	Ø.2199928	2.	Ø.1312834
3	Ø.2247317	Ø.ØØØ2Ø26	Ø.2245291	1.	Ø.ØØ45363
4	Ø.225Ø134	Ø.ØØØ2Ø26	Ø.22481Ø8	1.	Ø.ØØØ2817

THE ITERATIVE PROCESS STOPS BECAUSE THE CONVERGENCE TEST VALUE IS REACHED

DIMENSION	EIGENVALUE
1	Ø.5638
2	Ø.2523
3	Ø.Ø486
4	Ø.Ø345

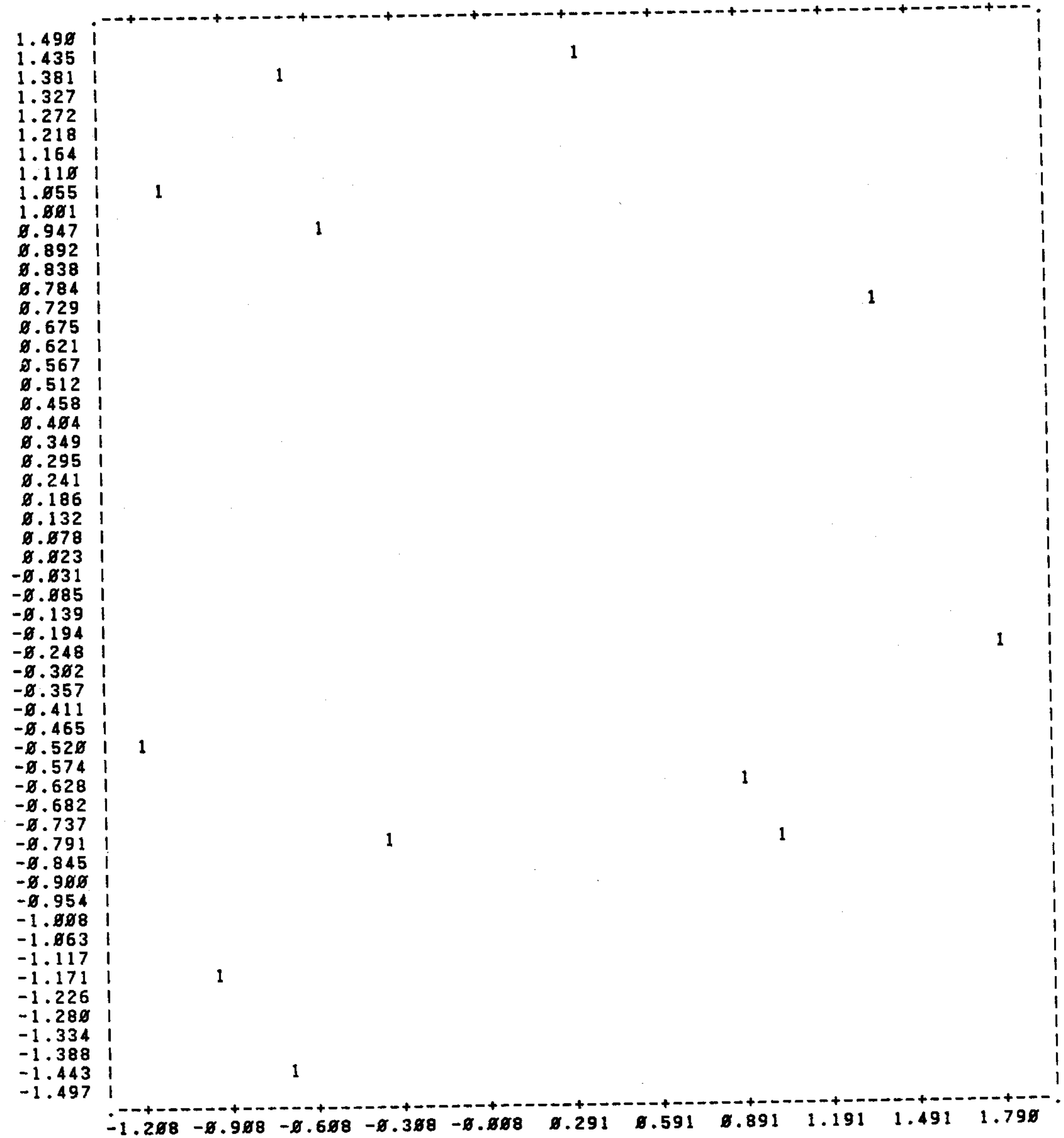
NUMBER OF ITERATIONS	TOTAL STRESS	SINGLE STRESS	TOTAL MULTIPLE STRESS			
			1	2	3	4
4	35.3687	31.698Ø	Ø.9337	Ø.9289	Ø.9333	Ø.8748

THE OBSERVATION SCORES ARE:

OBSERVATIONS \ DIMENSIONS	1	2	3	4
1	1.3658	0.7321	-0.4566	-2.0046
2	0.3562	1.4272	-0.2744	-0.9785
3	-1.1161	1.0766	0.0282	0.5040
4	-0.5470	0.9349	1.7236	0.4895
5	-0.6546	1.4073	-0.7018	1.0896
6	-0.9446	-1.1552	-0.5154	-1.4885
7	-1.2077	-0.5097	-1.5127	0.0585
8	-0.6717	-1.4460	-0.0646	0.1460
9	-0.3269	-0.8166	2.3078	-0.5840
10	1.7905	-0.2418	0.2121	0.6365
11	1.0331	-0.7780	-0.5224	0.8594
12	0.9232	-0.6307	-0.2238	1.2721

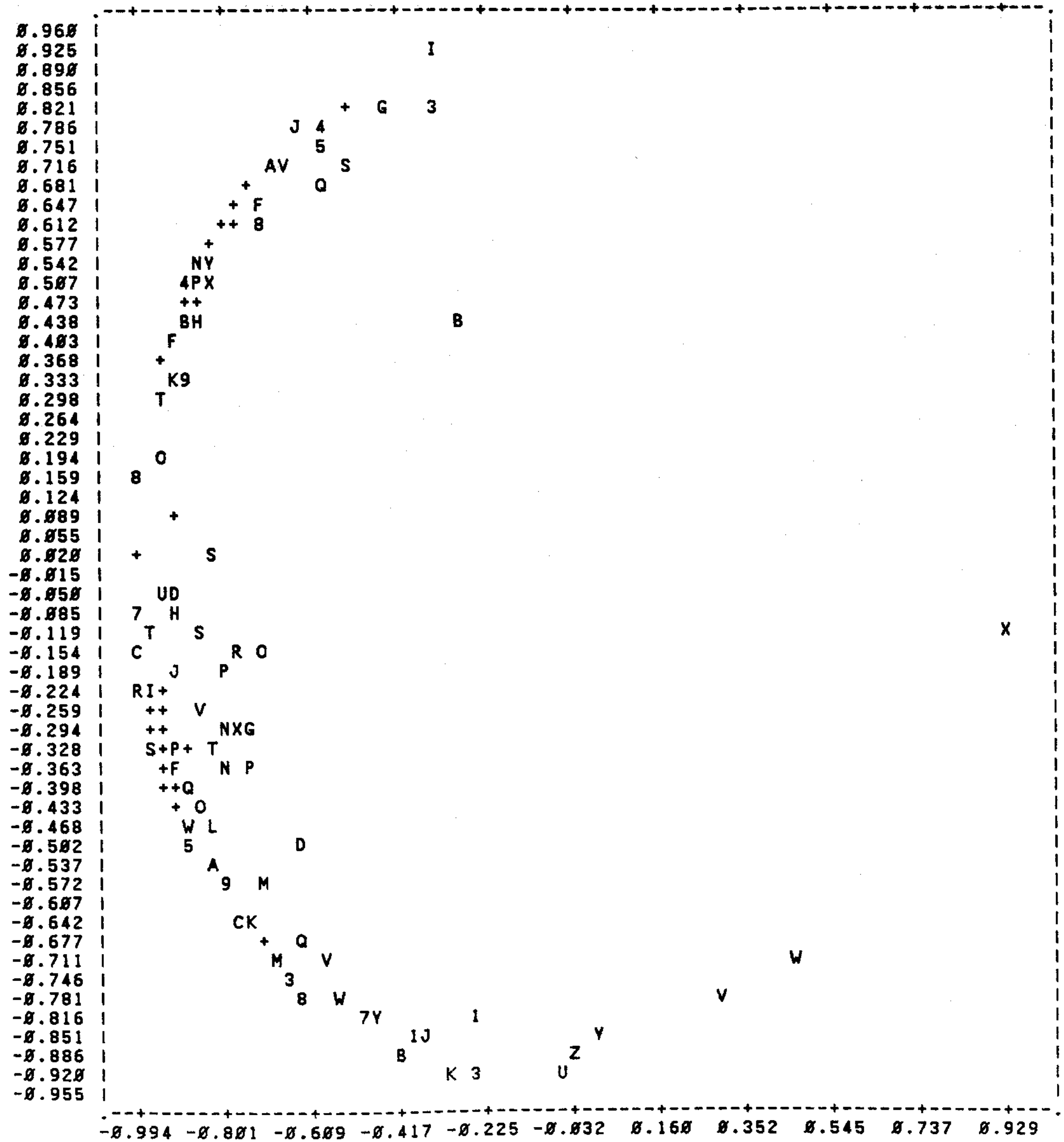
OBSERVATION SCORES, UNLABELED

PRINCALS WITH THE PARL68 DATA ORDINAL EN NUMERICAL RECODED



DIRECTION COSINES LABELED
BY VARIABLE NUMBER

PRINCALS WITH THE PARL68 DATA ORDINAL EN NUMERICAL RECODED



SUMMARY OF ALL CELLS (X,Y), MARKED : + IN THE PLOT, CONTAINING MORE THAN 1 POINT IDENTIFICATION

Y	X	NUMBER OF POINTS	POINT-IDENTIFICATION
Ø.821	-Ø.527	2	2U
Ø.681	-Ø.746	2	2A
Ø.647	-Ø.774	2	LE
Ø.612	-Ø.8Ø1	3	R5M
Ø.612	-Ø.774	4	1616
Ø.577	-Ø.829	7	3DC9C1Z
Ø.473	-Ø.884	3	QZ7
Ø.473	-Ø.856	2	8W
Ø.368	-Ø.939	2	O7
Ø.Ø89	-Ø.911	2	LM
Ø.Ø2Ø	-Ø.994	2	4D
-Ø.224	-Ø.939	2	BU
-Ø.259	-Ø.966	2	J6
-Ø.259	-Ø.939	3	2II
-Ø.294	-Ø.966	3	L9A
-Ø.294	-Ø.939	2	TZ
-Ø.328	-Ø.939	2	NE
-Ø.328	-Ø.884	2	HF
-Ø.363	-Ø.939	2	2G
-Ø.398	-Ø.939	2	E5
-Ø.398	-Ø.911	2	HR
-Ø.433	-Ø.911	5	KGX46
-Ø.677	-Ø.719	2	YE

THE HISTORY OF ITERATIONS TO COMPUTE THE FINAL CONFIGURATION

NUMBER OF ITERATIONS	TOTAL STRESS	MULTIPLE STRESS	SINGLE STRESS	AVERAGE NUMBER OF INNER ITERATIONS	DIFFERENCE BETWEEN THE LAST TWO ITERATIONS
1	Ø.2418488	Ø.ØØØ2Ø26	Ø.2416462	1.	Ø.Ø168354
2	Ø.2426946	Ø.ØØØ2Ø26	Ø.242492Ø	1.	Ø.ØØØ8458
3	Ø.243Ø912	Ø.ØØØ2Ø26	Ø.2428886	1.	Ø.ØØØ3965
4	Ø.2433326	Ø.ØØØ2Ø26	Ø.24313ØØ	1.	Ø.ØØØ2414
5	Ø.2435ØØ8	Ø.ØØØ2Ø26	Ø.2432982	1.	Ø.ØØØ1683
6	Ø.24362Ø9	Ø.ØØØ2Ø26	Ø.2434183	1.	Ø.ØØØ12ØØ
7	Ø.2437114	Ø.ØØØ2Ø26	Ø.2435Ø88	1.	Ø.ØØØØ9Ø5
8	Ø.2437824	Ø.ØØØ2Ø26	Ø.2435798	1.	Ø.ØØØØ71Ø
9	Ø.2438431	Ø.ØØØ2Ø26	Ø.24364Ø5	1.	Ø.ØØØØ6Ø7
1Ø	Ø.2438946	Ø.ØØØ2Ø26	Ø.243692Ø	1.	Ø.ØØØØ516
11	Ø.2439398	Ø.ØØØ2Ø26	Ø.2437372	1.	Ø.ØØØØ451

THE ITERATIVE PROCESS STOPS BECAUSE THE CONVERGENCE TEST VALUE IS REACHED

DIMENSION	EIGENVALUE
1	Ø.58Ø5
2	Ø.2613
3	Ø.Ø741
4	Ø.Ø59Ø

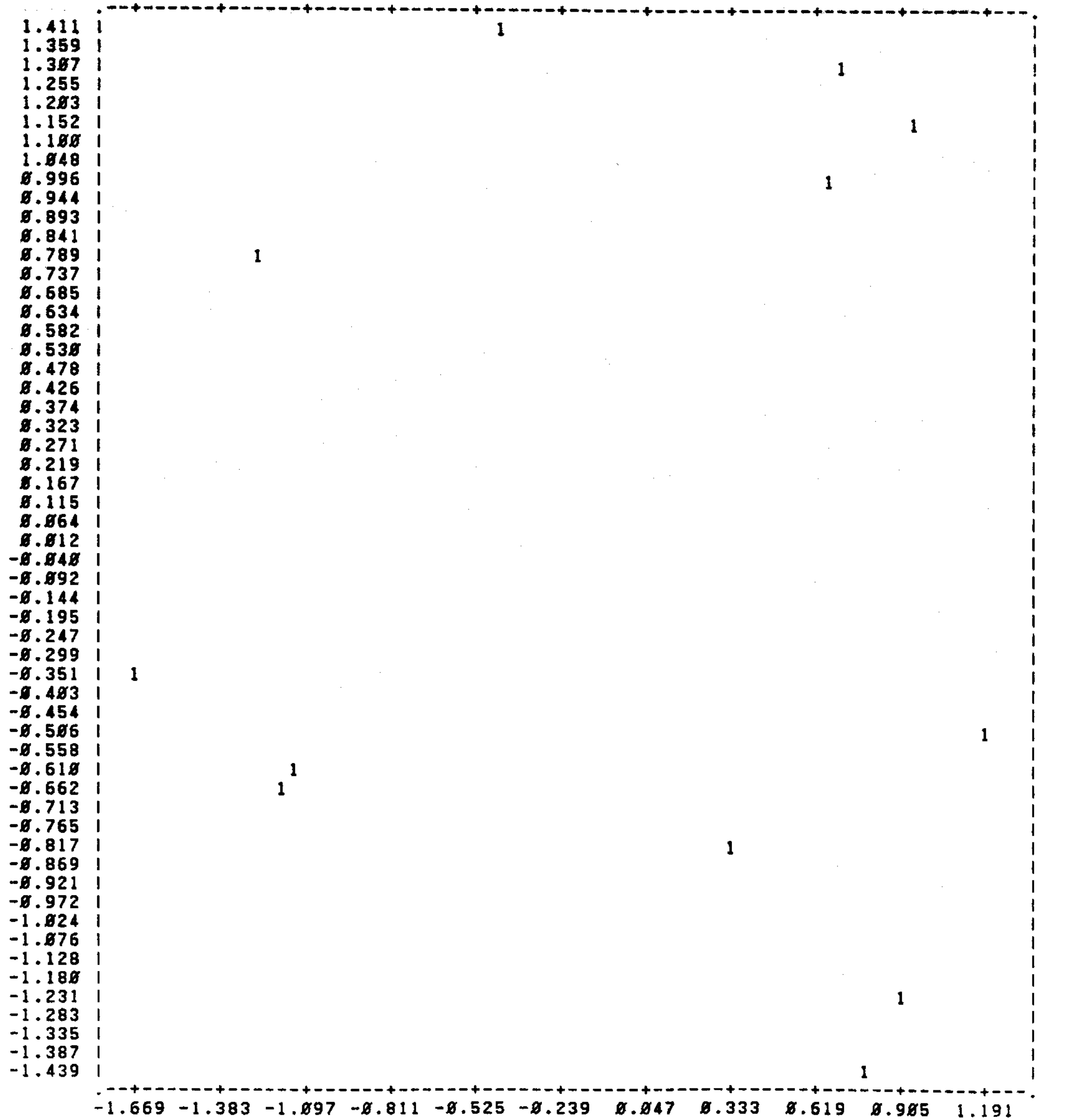
NUMBER OF ITERATIONS	TOTAL STRESS	SINGLE STRESS	TOTAL MULTIPLE STRESS			
			1	2	3	4
11	38.ØØ75	34.33Ø6	Ø.9336	Ø.93Ø2	Ø.9261	Ø.8872

THE OBSERVATION SCORES ARE:

```
        *
         * DIMENSIONS
          *
           *
            *       1         2         3         4
OBSERVATIONS *******************************************
           1 *   -1.2393    Ø.7762   -1.2577   -1.5322
           2 *   -Ø.4229    1.3945   -Ø.8515   -1.1838
           3 *    Ø.9649    1.1259    Ø.1875    Ø.73Ø3
           4 *    Ø.6696    1.ØØ74    1.692Ø   -Ø.2198
           5 *    Ø.7293    1.3164    Ø.Ø74Ø    1.2818
           6 *    Ø.9129   -1.2243   -Ø.8232   -Ø.71Ø7
           7 *    1.1914   -Ø.53ØØ   -1.53Ø2    Ø.7Ø99
           8 *    Ø.7954   -1.4334   -Ø.2Ø79   -Ø.Ø486
           9 *    Ø.342Ø   -Ø.8291    1.8599   -1.648Ø
          1Ø *   -1.6687   -Ø.338Ø    Ø.3929    Ø.9517
          11 *   -1.1448   -Ø.6521    Ø.1788    Ø.833Ø
          12 *   -1.1298   -Ø.6134    Ø.2853    Ø.8364
```

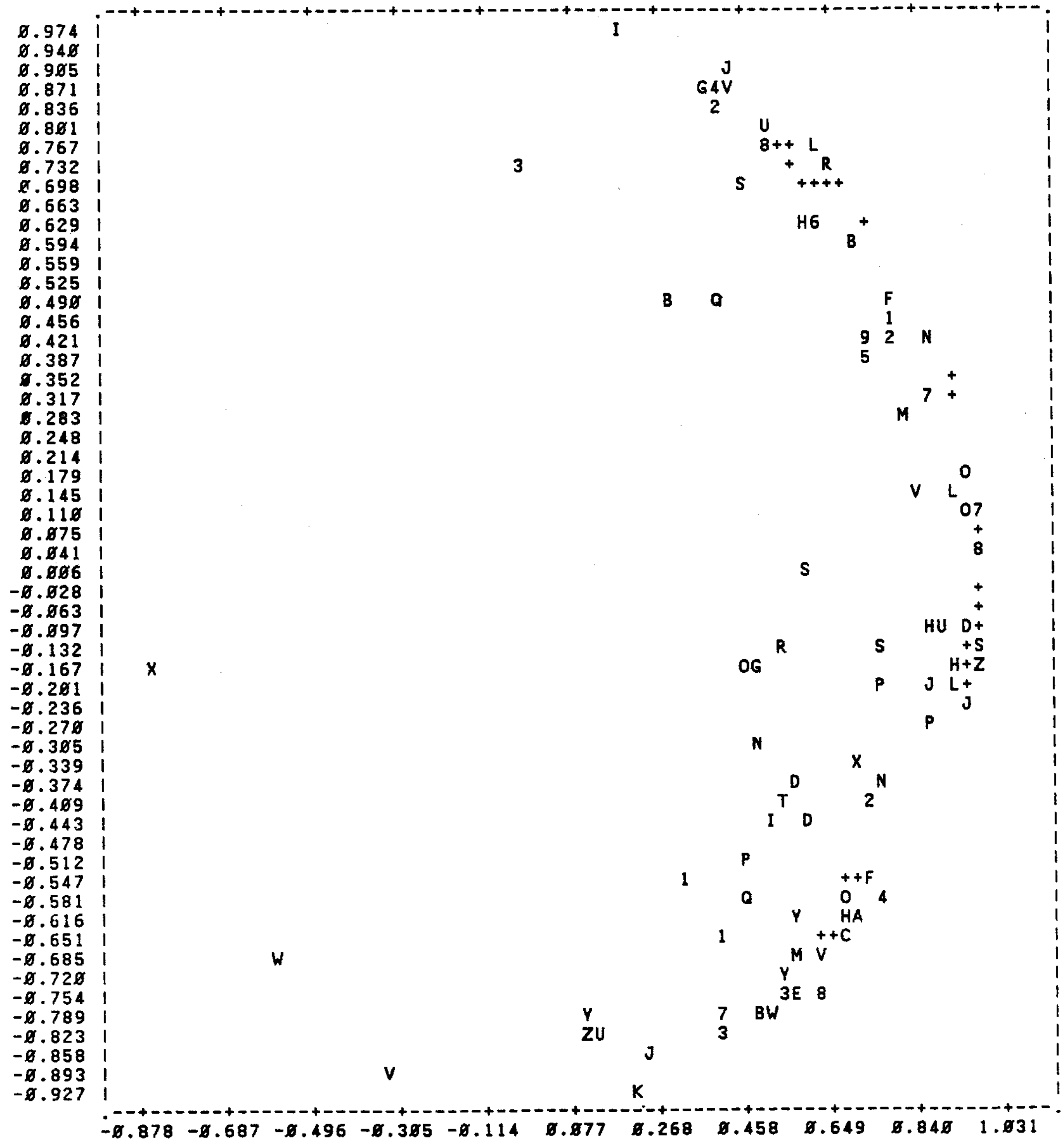
OBSERVATION SCORES, UNLABELED

PRINCALS WITH THE PARL68 DATA ORDINAL EN NUMERICAL RECODED



DIRECTION COSINES LABELED
BY VARIABLE NUMBER

PRINCALS WITH THE PARL68 DATA ORDINAL EN NUMERICAL RECODED



SUMMARY OF ALL CELLS (X,Y), MARKED : + IN THE PLOT, CONTAINING MORE THAN 1 POINT IDENTIFICATION

Y	X	NUMBER OF POINTS	POINT-IDENTIFICATION
Ø.767	Ø.54Ø	3	X3F
Ø.767	Ø.568	3	Y51
Ø.732	Ø.568	2	91
Ø.698	Ø.595	2	PW
Ø.698	Ø.622	2	ZM
Ø.698	Ø.649	2	DC
Ø.698	Ø.677	2	CA
Ø.629	Ø.731	3	A6E
Ø.352	Ø.922	2	48
Ø.317	Ø.922	2	ZQ
Ø.Ø75	Ø.977	2	TK
-Ø.Ø28	Ø.977	4	7BTE
-Ø.Ø63	Ø.977	2	UR
-Ø.Ø97	Ø.977	3	4CE
-Ø.132	Ø.949	5	2GI6I
-Ø.167	Ø.949	5	T59AR
-Ø.2Ø1	Ø.949	5	NX5KG
-Ø.547	Ø.677	2	LF
-Ø.547	Ø.7Ø4	2	QW
-Ø.651	Ø.622	2	9M
-Ø.651	Ø.649	2	6K

Job set up for Princals version 3.02 April 1979

Card 1 job number card

Column	Code	Format	Description
1-5	IJOB	I5	number of jobs

Card 2 title card

Column	Code	Format	Description
1-80	NAME	20A4	any alphameric code to name the analysis

Card 3 problem size parameter card

Column	Code	Format	Description
1-5	NOBS	I5	number of observation units
6-10	NVR1	I5	number of variables in the data matrix (≥ NVR2)
11-15	NVR2	I5	number of variables in the analysis
16-20	NDIM	I5	number of dimensions
21-25	MAXC	I5	greatest possible number of categories in the dataset within one variable
26-30	ISUC	I5	total number of categories

Card 4 analysis parameter card

Column	Code	Format	Description
1-5	MAXI	I5	maximum number of iterations to compute the final solution
6-15	EPSI	E10.8	stop criterion for the final solution
16-20	NITIN	I5	maximum number of iterations to compute the initial configuration
21-30	EPSIN	E10.8	stop criterion for the initial configuration

Card 5 I/O parameter card

The parameters IWR1, IWR2, IPLO, IOUT1, IOUT2, IOUT3 can be used for the initial configuration as well for the final solution. These parameters can be read with two formats, depending on the value of the parameter IPRIN: either IPRIN ≤ 1 and the parameters are read with the format I5, or IPRIN=2 and they are read with the format 3X,2I1. If IPRIN=2 the first column read applies to the initial configuration and the second column applies to the final solution.

Column	Code	Format	Description
1-5	INPU	I5	unit number of the input data
6-10	IDAT	I5	input data listing 0 = no 1 = yes
11-15	IWR1	I5 or (if IPRIN=2:3X,2I1	print options for quantifications initial) final 0 0 = no print 1 1 = print only the observation scores 2 2 = print the observation scores and the category scores 3 3 = print only the category scores
16-20	IWR2	I5 or (if IPRIN=2:3X,2I1	print the history of iterations initial) final 0 0 = no 1 1 = yes

Card 5 continued

Column	Code	Format	Description
21-25	IPLO	I5	plot options for quantifications
		(if IPRIN=2: 3X,2I1 initial) final	
		0	0 = no plots
		1	1 = plot the observation.scores and the direction cosines
		2	2 = like IPLO = 1 and: partitioned plots of observation scores and plot the category scores both according to the variables, specified in IPARTI
26-30	IOUT1	I5	unit nr. for the output of the observation scores to other media than the line printer
		(if IPRIN=2: 3X,2I1 initial) final	
		0	0 = no extra output required
		k	k = output to nr. k
31-35	IOUT2	I5	unit nr. for the output of the category scores to other media than the line printer
		(if IPRIN=2: 3X,2I1 initial) final	
		0	0 = no extra output required
		k	k = output to nr. k
36-40	IOUT3	I5	unit nr. for output of the scaled data vectors to other media than the line printer.
		(if IPRIN=2: 3X,2I1 initial) final	
		0	0 = no extra output required
		k	k = output to nr. k
41-45	ILEV	I5	measurement levels of variables
			0 = mixed levels which are specified in ICATGO
			1 = multiple nominal variables only
			2 = ordinal variables only
			3 = numerical variables only
46-50	IPRIN	I5	i/o options for the initial configuration (=i.c.
			0 = no output of the i.c.
			1 = identical options as for the final solution
			2 = options specified in the first column of the relevant parameters
51-55	ICAT	I5	number of categories per variable
			0 = variables have different numbers of categories
			k = all variables have k categories

ONLY IF ICAT = 0   Category numbers card(s)   (*16* vars per card)

Column	Code	Format	Description
1-80	ICATGO	***16I5***	the maximum numbers of categories of all variables

ONLY IF IPLO = 2 Card(s) specifying plot partitioning and category scores plot selecting variables (max. NVR1)   (80 vars. per card)

Column	Code	Format	Description
1-80	IPARTI	80I1	the columns specify the variables in the same order as in the data matrix
			IPARTI(i) = 0 = no extra plot options for this variable(i)
			IPARTI(i) = 1 = observation scores plot, labeled with categories of this variable(i)

Continuation of the plot partitioning and category scores plot selecting variables card

Column	Code	Format	Description
			IPARTI(i) = 2 = like IPARTI(i) = 1 and: the plot of the category scores of variable (i).
			IPARTI(i) = 3 = plot the category scores of variable(i)

ONLY IF ILEV = 0 measurement level for variable (i), i = 1,2,..,NVR2 (20 vars. per Card)

Column	Code	Format	Description
1-80	ITYP	20I5	ITYP(i) = 0 = variable(i) is multiple nominal
			" = 1 = variable(i) is single nominal
			" = 2 = variable(i) is single ordinal
			" = 3 = variable(i) is single numerical

Three Variable format cards; the last cards before the datamatrix

Column	Code	Format	Description
1-80	FMT	3(20A4)	variable integer format *(FORTRAN)*

The data matrix can follow now (depending on the value of INPU)

Depending on the value of IJOB more analyses can be done, all cards, except for the jobnumber card, have to be repeted hereafter. This has to be done for every extra job.