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CHI - SQUARE :

a short review

RB - 003 - 71

0 The multinomial distribution

Observations in m classes; frequencies $n_1, \ldots, n_m$; total frequency N.

Distribution (random sampling with replacement from a finite population, or with/without replacement from an infinite population):

$$p(n_1,\ldots,n_m \mid N) = N! \prod_{i=1}^{m} \frac{p_i^{n_i}}{n_i!} .$$

Some expected values:

$$E(n_i) = Np_i,$$

$$VAR(n_i) = E(n_i^2) - E^2(n_i) = Np_i(1-p_i),$$

$$COVAR(n_i,n_j) = E(n_i n_j) - E(n_i)E(n_j) = -Np_i p_j. \qquad (i \neq j)$$

1 Test of a simple hypothesis

1.1 Simple hypotheses

$$H_0\colon p_i = \hat{\beta}_i \qquad (i=1,\ldots,m),$$

$$H_1\colon p_i \neq \beta_i \qquad (\text{at least one } i),$$

(β_i completely specified). Define the new random variable

$$z_i = \frac{n_i - N\beta_i}{\sqrt{N\beta_i}} .$$

If H_0 is true, then

$$E(z_i) = 0,$$

$$VAR(z_i) = 1 - \beta_i,$$

$$COVAR(z_i,z_j) = -\sqrt{\beta_i \beta_j}. \qquad (i \neq j)$$

1.2 Asymptotic distribution

If $N\beta_i \to \infty$ for all i then by the multivariate Moivre-Laplace theorem the distributio of the z_i tends to an m-variate normal distribution with means zero, variances

$$\gamma_{ii} = 1 - \beta_i,$$

and covariances

$$\gamma_{ij} = -\sqrt{\beta_i \beta_j} \qquad (i \neq j).$$

1.3 Orthogonal systems

Consider m vectors $y^0, y^1, \ldots, y^{m-1}$ with m elements each, satisfying

$$\sum_{i=1}^{m} y_i^s y_i^t = \delta^{st}, \qquad (s,t=0,\ldots,m-1)$$

and

$$y_i^0 = \beta_i^{\frac{1}{2}}. \qquad (i=1,\ldots,m).$$

Define the m new random variables

$$x_s = \sum_{i=1}^{m} y_i^s z_i . \qquad (s=0,\dots,m-1)$$

Obviously $x_0 = 0$, no matter what n_i is. For $s=1,\dots,m-1$ we find that x_s is asymptotically normally distributed with mean zero. The covariance of x_s and x_t is

$$\begin{aligned}
\ell_{st} &= \sum_{i=1}^{m} \sum_{j=1}^{m} y_i^s y_j^t \gamma_{ij} = \\
&= \sum_{i=1}^{m} y_i^s y_i^t \gamma_{ii} + \sum_{\substack{i=1 \\ (i \neq j)}}^{m} \sum_{j=1}^{m} y_i^s y_j^t \gamma_{ij} = \\
&= \sum_{i=1}^{m} y_i^s y_i^t (1 - \beta_i) - \sum_{\substack{i=1 \\ (i \neq j)}}^{m} \sum_{j=1}^{m} y_i^s y_j^t \sqrt{\beta_i \beta_j} = \\
&= \sum_{i=1}^{m} y_i^s y_i^t - \sum_{i=1}^{m} \sum_{j=1}^{m} y_i^s y_j^t \sqrt{\beta_i \beta_j} = \\
&= \delta^{st} - \sum_{i=1}^{m} y_i^s \sqrt{\beta_i} \sum_{j=1}^{m} y_j^t \sqrt{\beta_j} = \delta^{st},
\end{aligned}$$

for $s,t=1,\dots,m-1$. Thus x_0 is a trivial random variable which assumes the value zero with probability one, and $x_1,\dots,x_{m-1}$ are asymptotically uncorrelated (and thus independent) standardized normal variates.

1.4 X^2-statistic

We have

$$\begin{aligned}
\sum_{s=0}^{m-1} x_s^2 &= \sum_{s=0}^{m-1} \Big(\sum_{i=1}^{m} y_i^s z_i\Big)^2 = \sum_{s=0}^{m-1} \sum_{i=1}^{m} \sum_{j=1}^{m} y_i^s y_j^s z_i z_j = \\
&= \sum_{i=1}^{m} \sum_{j=1}^{m} z_i z_j \sum_{s=0}^{m-1} y_i^s y_j^s = \sum_{i=1}^{m} \sum_{j=1}^{m} z_i z_j \, \delta^{ij} = \\
&= \sum_{i=1}^{m} z_i^2 .
\end{aligned}$$

And thus, if $N\beta_i \to \infty$ for all i and H_0 is true, the statistic

$$X^2 = \sum_{i=1}^{m} z_i^2 = \sum_{i=1}^{m} \frac{(n_i - N\beta_i)^2}{N\beta_i} ,$$

is distributed as the sum of squares of m-1 independent standardized normal variates with zero expectations, i.e. as χ^2 with m-1 degrees of freedom.

1.5 Partitioning X^2

It also follows that x_s^2 is distributed asymptotically as χ^2 with one dfr for all $s=1,\dots,m-1$; that $x_s^2 + x_t^2$ is distributed asymptotically as χ^2 with two dfr for all $s,t=1,\dots,m-1$ with $s \neq t$, and so on. Because there is an infinite number of ways to choose the vectors y, there also is an infinite number of ways to partition X^2. We must however always choose the y without reference to the data, because otherwise the y_i^s are not constants any more but random variables and our derivation of the limiting distribution of x_s^2 is invalid.

1.6 Between-Within contrasts

The most important choices for y are those for which the components of X^2 (i.e. x_1^2, x_2^2, etc) are actual X^2 values for component vectors of observed frequencies. These are the between-within contrasts in which we leave out certain classes and/or poole others. If this is done in the usual orthogonal fashion (k groups, k within-group X^2 values, and one between-group X^2 value) we have an additive partition of X^2. To preserve additivity we must use modified X^2 values for the within-group contrasts, and not the (asymptotically equivalent) actual X^2 values.

2 Test of a composite hypothesis

In the composite case the null hypothesis is

$$H_0: p_i = \beta_i(\theta). \qquad (i=1,\dots,m)$$

The probabilities β_i are predescribed functions of r free parameters $\theta_1,\dots,\theta_r$. We are interested in the asymptotic distribution of

$$X^2(\theta) = \sum_{i=1}^{m} \frac{(n_i - N\beta_i(\theta))^2}{N\beta_i(\theta)}$$

if we estimate the parameters θ. Under some regularity conditions on the functions $\beta_i(\theta)$, it can be proved that if we substitute the maximum likelihood estimates $\hat{\theta}$ for θ, then the statistic again has an asymptotic χ^2 distribution under H_0 with m - r dfr. We replace the given number β_i in our previous development by $\beta_i(\hat{\theta})$, and our whole discussion again applies.

3 The contingency table

3.1 Composite hypothesis of independence

Notation $N = \{n_{ij}\}$ $(i=1,\dots,n;\ j=1,\dots,m)$. Marginals $n_{i.}$, $n_{.j}$, grand total $n_{..}$. Suppose $m \leq n$. The same discussion applies, with slight modifications. The most interesting hypothesis is complete independence

$$H_0: p_{ij} = \alpha_i \beta_j.$$

Maximum likelihood estimates

$$\hat{\alpha}_i = n_{i.}/n_{..},$$

$$\hat{\beta}_j = n_{.j}/n_{..},$$

and

$$X^2(\hat{\alpha}_i,\hat{\beta}_j) = \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{(n_{ij} - n_{..}\hat{\alpha}_i\hat{\beta}_j)^2}{n_{..}\hat{\alpha}_i\hat{\beta}_j} = n_{..}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{n_{ij}^2}{n_{i.}n_{.j}} - 1\right]$$

is distributed (H_0 true, $n_{i.}n_{.j}/n_{..} \to \infty$ for all i, j) as χ^2 with $nm - (n + m - 1) = (n - 1)(m - 1)$ dfr.

In this case we need two sets of orthonormal vectors in stead of our single set $y^0, \ldots, y^{m-1}$. These two sets $v^0, \ldots, v^{m-1}$ and $w^0, \ldots, w^{n-1}$ must satisfy

$$\sum_{i=1}^{n} \hat{\alpha}_i w_i^s w_i^t = \delta^{st},$$

$$\sum_{j=1}^{m} \hat{\beta}_j v_j^s v_j^t = \delta^{st},$$

and

$$w_i^0 = \hat{\alpha}_i^{\frac{1}{2}},$$

$$v_j^0 = \hat{\beta}_j^{\frac{1}{2}}.$$

The nm new rv's x_{st} are defined by

$$x_{st} = \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} w_i^s v_j^t,$$

and consequently $x_{00} = 0$, but also $x_{s0} = 0$ for all $s=1,\ldots,n$, and $x_{0t} = 0$ for all $t=1,\ldots,m-1$. Thus

$$X^2 = \sum_{s=1}^{n-1} \sum_{t=1}^{m-1} x_{st}^2$$

is distributed as χ^2 with $(n-1)(m-1)$ dfr.

3.2 Canonical partition

Standard results in the theory of canonical forms of matrices tell us that we can choose v and w in such a way that $x_{st} = 0$ for all $s \neq t$, and for $s > m$. There are only m-1 components of X^2 not equal to zero. With different notation we can say

$$X^2 = n_{..} \sum_{s=1}^{m-1} \lambda_s^2,$$

and

$$n_{ij} = \frac{n_{i.} n_{.j}}{n_{..}} \left(1 + \sum_{s=1}^{m-1} \lambda_s w_i^s v_j^s\right).$$

The λ_s^2 can be interpreted as squared canonical correlations.

3.3 Between-Within contrasts

In the same way as before it is possible to form between and within contrasts, now both for rows and columns. Using these we can partition N, for example, into $(n-1)(m-1)$ twofold tables with 1 dfr each, whose modified X^2 values add up to the total X^2.

4 Posterior testing

If we form our orthonormal functions after inspection of the data we clearly need a different testing procedure. The theory of posterior testing tells us to test any component of the total X^2 as if it was a χ^2 with $(n-1)(m-1)$ dfr. It can be shown that with this strategy the probability that a true hypothesis is rejected is less than the significance level chosen.

5 Literature

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F. Yates The analysis of contingency tables with groupings based on quantitative characters
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6 Examples

6.1 Artificial example. Equiprobability test, six classes, 120 observations. $e_i = N\hat{p}_i = 20$ for all i. Example of a between-within analysis, three groups, each of size two.

6.2 Example from Lammers. Symmetric two-way within-between analysis of transition frequencies father's political choice - student's political choice (University of Amsterdam 1964). 5 x 5 input table somewhere in the middle of the page.

6.3 Own Example. Number of children from environment A-B-C-D-E which goes to school form A-B-C-D-E (columns) after leaving 6th grade. Environment (father's profession): A academic, director, etc. B: high white collar, army officers. C: Shop keepers, low white collar, D: Schooled labor, E: Unskilled labor. Two-way within-between.

6.4 Main example from Lammers. First page: input three-way contingency table. Student political choices per university per faculty. Second, third, and fourth page: two-way marginals. Fifth page: canonical partition of the two main two-way marginals. Sixth page: plot of out-put on fifth page. Seventh page: Lammers used prior scores for left-right

CONF	1
VVD	0
PVDA	3
PACO	4
D'66	2

On this page they are compared with the canonical scores. Eighth page: compute all possible within components for parties and investigate possible governmental coalitions for homogeneity. Nineth page: three-way joint bivariate canonical analysis.

n_i	e_i	n_i-e_i	$(n_i-e_i)^2$		y^0	y^1	y^2	y^3	y^4	y^5
10	20	−10	100		1	1	0	0	1	1
12	20	− 8	64		1	−1	0	0	1	1
19	20	− 1	1		1	0	1	0	−2	0
23	20	3	9		1	0	−1	0	−2	0
29	20	9	81		1	0	0	1	1	−1
27	20	7	49		1	0	0	−1	1	−1
120	120	0	304		(120)	(40)	(40)	(40)	(240)	(80)
				x_s^2	0	.1	.4	.1	.15	14.45

Between groups

n_i	e_i	n_i-e_i	$(n_i-e_i)^2$
22	40	−18	324
42	40	2	4
56	40	16	256
120	120	0	584

$X^2 = 584/40 = 14.6 = x_4^2 + x_5^2.$

Within group I

n_i	e_i	n_i-e_i	$(n_i-e_i)^2$
10	20	−10	100
12	20	− 8	64
22	40	−18	324

$\overline{X}^2 = 2/11 = .1818.$

$X^2 = 164/20 - 324/40 = .1 = x_1^2.$

In the same way: within group II $X^2 = .4 = x_2^2.$ $\overline{X}^2 = .2195.$

within group III $X^2 = .1 = x_3^2.$ $\overline{X}^2 = .0714.$

Additive partition of X^2

Source	X^2	dfr	P
Within groups	.6	3	.896
Between groups	14.6	2	<.001
Total	15.2	5	.009

Asymptotic partition

$\overline{X}^2$	dfr	P
.5327	3	.903
14.6	2	<.001
15.1327	5	.009

VVD	KVP	AR/CH	PVDA	PSP/CPN	
-1	1	1	-1	-1	demominational/non
-1	0	0	1	1	left/right
0	0	0	1	-1	pink/red
0	1	-1	0	0	cathOlic/protestant

78.574	0.292	0.005	6.219	85.090
0.000	67.456	0.764	0.000	68.220
0.047	0.046	7.900	0.118	8.111
16.877	4.537	0.107	53.983	75.504
95.498	72.331	8.776	60.320	236.925

39	08	73	15
00	05	04	04
(1,4)		(3,3)	
40	36	26	10
07	27	12	15
(4,1)		(4,2)	

51	00	02	17	07
10	39	01	21	05
15	00	07	09	03
14	00	03	73	15
01	00	00	04	04

Diagonal partition

Stability	207.913	dfr = 4	
Change	29.012	dfr =12	P = .004
Total	236.925	dfr =16	

Canonical partition

Component I	135.312	101.614	7.346	1E-13	(catholics-noncatholics)
Component II	72.541	29.074	1.987	.022	(lib & prot-pink & red)
Component III	21.243	7.831	-1.680	.954	(protestants-liberals)
Component IV	7.830				(pink-red)
Total	236.926		12.469		

	A	B	C	D	E	
A	28	00	00	00	00	28
B	29	00	00	00	00	29
C	45	06	04	01	00	56
D	49	36	07	10	03	105
E	03	09	04	07	03	26
	154	51	15	18	06	244

A = HAVO, MAVO, VWO.

B = LTS, LHNO.

C = LAVO, LEAO.

D = ITO, INOM, MEL.

E = STOP.

01	+1 +1 +1 −1 −1					+1 −1 −1 −1 −1					66.646	66.646
02	+1 +1 +1 −1 −1					0 +1 +1 −1 −1					0.854	1.980
03	+1 +1 +1 −1 −1					0 +1 −1 +1 −1					0.513	1.189
04	+1 +1 +1 −1 −1					0 +1 −1 −1 +1					0.180	0.417
05	+1 −1 0 0 0					+1 −1 −1 −1 −1					0.000	
06	+1 −1 0 0 0					0 +1 +1 −1 −1					0.000	
07	+1 −1 0 0 0					0 +1 −1 +1 −1					0.000	
08	+1 −1 0 0 0					0 +1 −1 −1 +1					0.000	
09	+1 +1 −1 0 0					+1 −1 −1 −1 −1					4.682	12.404
10	+1 +1 −1 0 0					0 +1 +1 −1 −1					0.467	
11	+1 +1 −1 0 0					0 +1 −1 +1 −1					0.280	
12	+1 +1 −1 0 0					0 +1 −1 −1 +1					0.098	
13	0 0 0 +1 −1					+1 −1 −1 −1 −1					11.046	10.743
14	0 0 0 +1 −1					0 +1 +1 −1 −1					8.070	3.244
15	0 0 0 +1 −1					0 +1 −1 +1 −1					2.675	1.527
16	0 0 0 +1 −1					0 +1 −1 −1 +1					4.263	2.174

Environment	X^2	dfr	P
Within ABC	5.527	8	.30
Within DE	26.054	4	$<.01$
Between ABC–DE	68.193	4	$<.01$
Total	99.774	16	$<.01$
School			
Within A	0.000	0	—
Within BCDE	17.400	12	.14
Between A–BCDE	82.374	4	$<.01$
Total	99.774	16	$<.01$

	LEIDEN	A'DAM VU	A'DAM GU	UTRECHT
JUR	13 32 09 02 18	01 00 00 01 01	03 14 12 02 10	07 16 01 00 05
MED	03 20 07 04 10	07 01 03 01 02	04 06 04 07 18	06 11 09 02 11
W&N	02 15 04 05 15	06 00 01 00 01	01 04 13 04 19	10 14 16 07 21
SOC	05 12 04 03 14	02 00 01 00 02	01 08 28 11 19	06 04 05 10 11
LET	05 08 06 03 05	02 00 00 01 00	07 08 16 08 16	03 10 04 04 13
TEC				
PSW			01 01 05 00 04	
VEE				03 06 01 01 05
TND				00 03 00 00 01
THE	01 01 03 02 00	05 00 00 00 00	01 00 00 00 00	05 00 00 02 01
LBW				
CIF	00 00 01 02 00	00 00 00 00 01	02 02 06 05 07	00 01 00 01 04
ECO		04 00 00 00 02	07 16 08 06 20	

	WAGENINGEN	GRONINGEN	DRIENENOORD	NIJMEGEN
JUR		02 08 06 00 07		05 07 03 01 06
MED		03 07 03 02 11		06 09 03 02 05
W&N		07 05 09 02 12		07 02 02 04 06
SOC	00 01 01 00 00	04 05 11 06 10		04 02 05 04 22
LET		01 05 06 02 06		04 00 08 02 04
TEC			00 07 03 04 05	
PSW				
VEE				
TND		01 02 00 01 01		01 02 00 00 02
THE		01 00 02 00 01		06 00 00 00 05
LBW	11 14 07 06 14			
CIF		00 01 01 01 00		00 00 01 00 00
ECO	00 00 00 00 01	01 13 05 00 07		

	TILBURG	ROTTERDAM	EINDHOVEN	DELFT
JUR	00 00 01 00 02	02 02 01 00 00		
MED		08 07 01 01 07		
W&N				
SOC	05 01 00 03 06	01 01 03 01 01		
LET				
TEC			12 07 03 00 13	24 66 22 20 50
PSW				
VEE				
TND				
THE				
LBW				
CIF				
ECO	03 11 02 00 09	09 33 15 02 16		

	JUR	MED	W&N	SOC	LET	TEC	PSW	VEE	TND	THE	LBW	CIF	ECO	TOT
DELFT						182								182
EINDH						035								035
RODAM	005	024		007									075	111
TBURG	003			015									025	043
NYGEN	022	025	021	037	018				005	011		001		140
DRIEN						019								019
GRONI	023	026	035	036	020				005	004		003	026	178
WAGEN				002							052		001	055
UTREC	029	039	068	036	034			016	004	008		006		240
ADAMG	041	039	041	067	055		011			001		022	057	334
ADAMV	003	014	008	005	003					005		001	006	045
LEIDE	074	044	041	038	027					007		003		234
TOTAAL	200	211	214	243	157	236	011	016	014	036	052	036	190	1616

	conf	vvd	pvda	cpn psp	d'66	tot
DELFT	24	66	22	20	50	182
EINDH	12	07	03	00	13	035
RODAM	20	43	20	04	24	111
TBURG	08	12	03	03	17	043
NYGEN	33	22	22	13	50	140
DRIEN	00	07	03	04	05	019
GRONI	20	46	43	14	55	178
WAGEN	11	15	08	06	15	055
UTREC	40	65	36	27	72	240
ADAMG	27	59	92	43	113	334
ADAMV	27	01	05	03	09	045
LEIDE	29	88	34	21	62	234
TOTAAL	251	431	291	158	485	1616

	conf	vvd	pvda	opn psp	d'66	tot
[illegible]JR	33	79	33	06	49	200
MED	37	61	30	19	64	211
MAN	33	40	45	22	74	214
SOC	28	34	58	38	85	243
LET	22	31	40	20	44	157
TEC	36	80	28	24	68	236
PSW	01	01	05	00	04	011
VEE	03	06	01	01	05	016
TND	02	07	00	01	04	014
THE	19	01	05	04	07	036
LBW	11	14	07	06	14	052
CIF	02	04	09	09	12	036
ECO	24	73	30	08	55	190
TOT	251	431	291	158	485	1616

	I	II
[illegible]	.029	-.001
[illegible]	-.007	-.017
[illegible]	-.008	.014
[illegible]	-.007	.007
[illegible]	-.001	.005
[illegible]	-.003	-.012
[illegible]	.028	-.003
[illegible]	.002	-.015
[illegible]	.007	-.007
[illegible]	.012	.009
[illegible]	-.022	-.007
[illegible]	-.006	.005
[illegible]	.006	-.003
[illegible]	.002	-.002
[illegible]	-.010	.016
[illegible]	.059	.011
[illegible]	-.005	-.013
LAMBDA	.070	.041
CHI	114	66
PPOST	.01	.05

	I	II
CONF	-.002	.024
VVD	-.018	-.005
PVDA	.012	-.005
PACO	.020	-.002
D'66	.004	-.004
JUR	-.016	-.000
MED	-.004	.003
W&N	.008	.000
SOC	.017	-.005
LET	.010	-.002
TEC	-.008	-.000
PSW	.019	-.013
VEE	-.016	.005
TND	-.027	-.002
THE	.012	.062
LBW	-.002	.009
CIF	.027	-.013
ECO	-.014	-.006
	.067	.030
	108	48
	.01	.50

Canonical partition universities

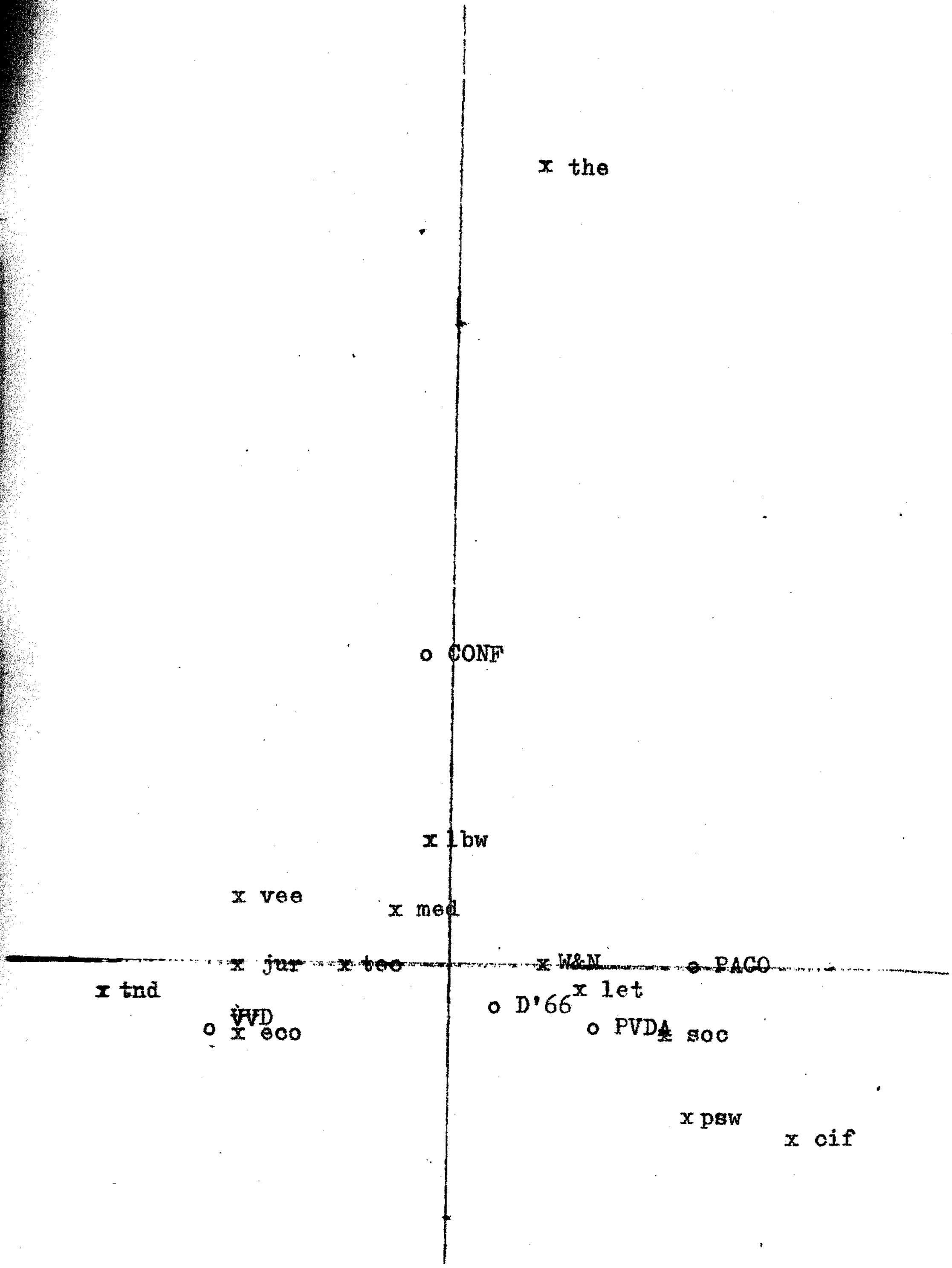
	X^2	Residual	Z	P
Component I	113.809	89.124	3.803	.00007
Component II	66.472	22.653	-2.722	.99677
Residual	22.647			
Total	202.928		9.4222	1E-21

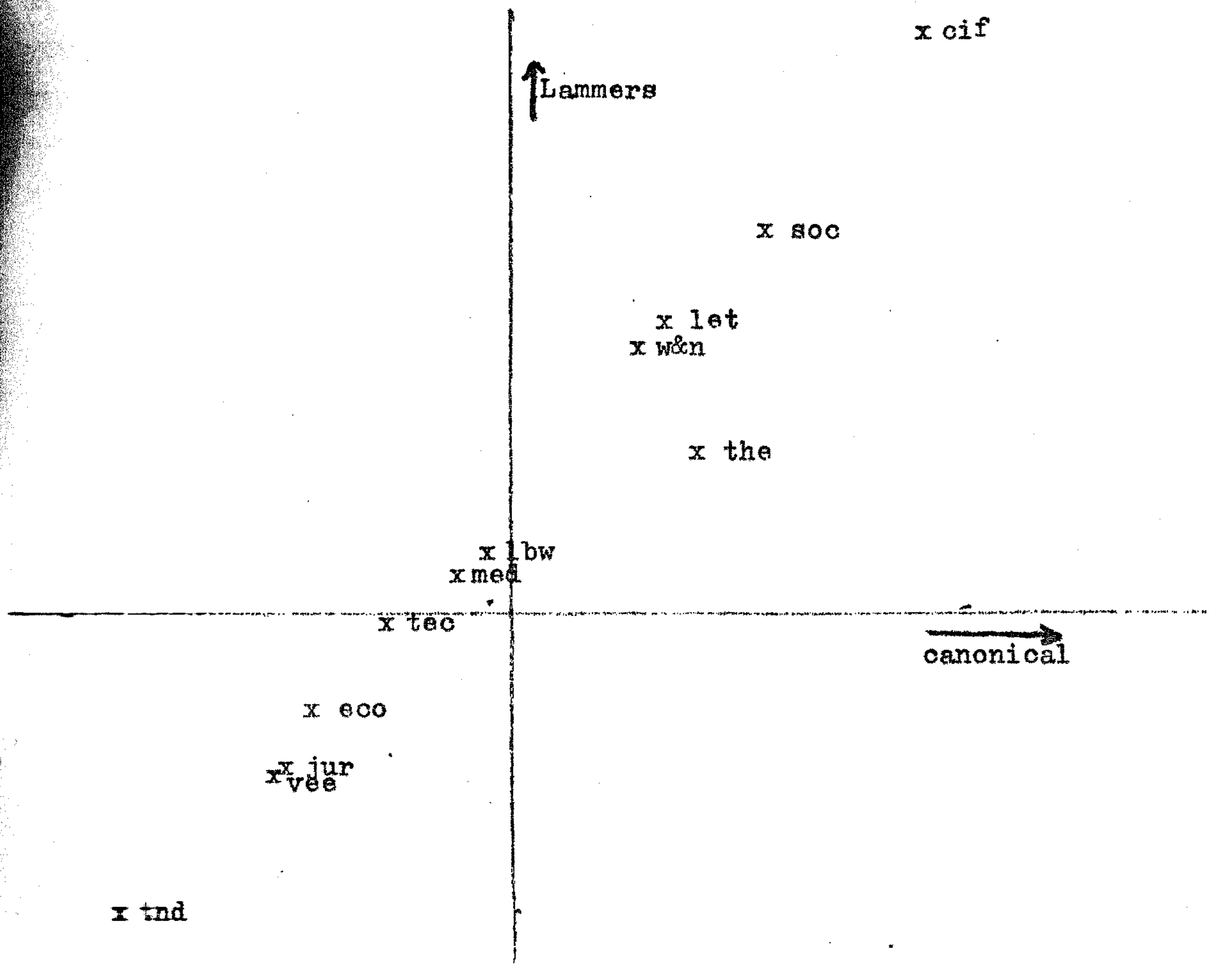
Canonical partition faculties

Component I	108.05	69.60	2.007	.02241
Component II	47.87	21.73	-3.343	.99958
Residual	21.73			
Total	177.65		8.1052	1E-16

... PLOT

... AND PARTIES





Lammers partition faculties

Source	X^2	dfr	F	P
Regression	105.546	12	4.419	.001
Residual	72.109	36		
Total	177.655	48		

Lammers partition universities

Source	X^2	dfr	F	P
Regression	61.656	11	1.311	.25
Residual	141.272	33		
Total	202.928	48		

[illegible] COMPONENTS

01111	129.844	00111	35.516	00011	21.097
10111	88.220	01011	97.776	00101	13.688
11011	143.206	01101	90.514	00110	20.010
11101	138.063	01110	121.637	01001	50.228
11110	166.642	10011	66.903	01010	77.142
		10101	62.489	01100	72.881
		10110	74.176	10001	38.628
		11001	96.771	10010	43.558
		11010	121.124	10100	46.354
		11100	119.461	11000	56.440

SELECTED COALITIONS

	X^2	dfr
10111	88.220	36
01000	0.000	0
	89.435	12
	177.655	48

	X^2	dfr
00111	35.516	24
11000	56.440	12
	85.699	12
	177.655	48

	X^2	dfr
11000	56.440	12
00101	13.688	12
00010	0.000	0
	107.527	24
	177.655	48

	X^2	dfr
10100	46.354	12
01001	50.228	12
00010	0.000	0
	81.063	24
	177.655	48

	X^2	dfr
10110	74.176	24
01001	50.228	12
	47.251	12
	177.655	48

	X^2	dfr
11001	96.771	24
00110	20.010	12
	60.874	12
	177.655	48

	I	II	III	IV	V	VI	VII
[redacted]					-.023	-.018	
[redacted]				-.011			
[redacted]			-.011			.012	
[redacted]							-.021
[redacted]				.011			
[redacted]	.038						
[redacted]				.057	.020	-.020	.060
[redacted]			-.019	-.022	-.018	.110	.017
[redacted]				-.019		.018	-.068
[redacted]			-.018	-.039	.042	-.017	
[redacted]		.087					
[redacted]				.034	.011		.026
[redacted]			.039				
[redacted]	.038						
[redacted]	.038				.017		
[redacted]			.043	-.011			
[redacted]			.034		.012		-.045
[redacted]			-.012		.013	-.012	-.022
[redacted]	.038			.012			
[redacted]							-.015
[redacted]		.084					
[redacted]			-.012			.029	
[redacted]				.020			.010
[redacted]				-.036	.043	-.013	.031
[redacted]					-.022	-.017	
[redacted]				-.021	.017		
[redacted]					-.016		
[redacted]				.013			
[redacted]				.012			
[redacted]							
[redacted]	.672	.658	.539	.480	.451	.422	.397
[redacted]	833	767	308	156	101	57	29
[redacted]	.000	.000	.002	1.000	1.000	1.000	1.000