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FINITE DECISION

LEARNING

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Finite decision learning experiments

1 Definition

1.1 FDL experiments* are nonasymptotic identification experiments in the sense of Luce, Bush, and Galanter (1963).

1.2 The basic elements of an FDL experiment are a finite stimulus set S, a finite set of hypotheses H, a probability distribution π on S x H, and a real valued loss function μ on H x H.

1.3 The instruction of an FDL experiment explains the nature of the sets S and H, and the nature of the function μ.

1.4 A block of size N is the result of N repeated independent trials on S x H according to $\pi(s,h)$.

1.5 In a training-run of lenght N we use a block of size N. We only show the subject the first coordinate of each (s,h) in the block. The subject responds with a particular $h' \in H$, and we give him the pay-off $\mu(h,h')$.

1.6 In a test-run of lenght N we also use a set of N elements from S x H (not necessarily a block), we also show the subject the first coordinates only, we also record the responses, but we do not give pay-offs.

1.7 The difference between test and training-runs and the random elements involved in the construction of blocks are explained to the subject in the instruction.

1.8 The expected pay-off of response $h' \in H$, given stimulus $s \in S$, is $\mathcal{E}(h' | s) = \sum \{\mu(h,h')\pi(h | s) \mid h \in H\}$.

1.9 A response $h' \in H$ is optimal, given stimulus $s \in S$, if $\mathcal{E}(h' | s) \geq \mathcal{E}(h | s)$ for all $h \in H$.

1.9 An FDL experiment is multinomial (or: an MFDL experiment) if there is a finite set T, a probability distribution λ on S x T, and a positive integer K such that each $h \in H$ is a function on T whose values are positive integers and which satisfies $\sum\{h(t) \mid t \in T\} =$

K. Moreover we require

$$\pi(h \mid s) = K! \prod_{t \in T} \frac{[\lambda(t \mid s)]^{h(t)}}{h(t)!} .$$

1.10 The <u>data</u> of an FDL experiment is a matrix $n(s,h)$ indicating how many times the subject has responded $h \in H$ when presented with $s \in S$ in a test-run of lenght N. Thus $\sum \{ n(s,h) \mid s \in S,\ h \in H \} = N$

2 <u>Particular cases</u>

2.1 In <u>classical paired associate learning</u> S and H have the same numbe of elements and there is a one-to-one map ϕ of S onto H such that $\pi(s,h) \neq 0$ iff $\phi(s) = h$.

2.2 In <u>classical concept learning</u> S has more elements than H, and ther is a map ψ of S onto H such that $\psi(s,h) \neq 0$ iff $\psi(s) = h$.

2.3 In <u>probabilistic classificatory concept learning</u> (De Klerk 1968) again S has more elements than H. For each $h \in H$ there is a probability distribution $\lambda_h(s)$ on S, and each $h \in H$ has a prior probability $\lambda(h)$. Define $\pi(s,h) = \lambda(h)\lambda_h(s)$.

2.4 In <u>probabilistic scalar concept learning</u> (De Klerk 1968) we are really dealing with a MFDL experiment with a somewhat unfortunate instruction.

2.5 In <u>functional learning</u> (De Klerk, De Leeuw, Oppe 1970) both S and H are sets of real numbers and there is a <u>smooth</u> function $\phi : S \to H$ This, and any other, definition of functional learning runs into difficulties if we try to specify what we mean by <u>smooth</u>.

2.6 In <u>multiple probability learning</u> (Vlek 1969) we are dealing with the degenerate special case in which S has only one element.

3 <u>Assumptions: set I</u>

3.1 The subjects generates (during training) a <u>subjective probability distribution</u> p on S x H and a real <u>utility function</u> u on H x H.

3.2 If we present the subject with stimulus $s \in S$ he computes for each $h' \in H$ the <u>subjectively expected utility</u> $E(h' \mid s) = \sum \{ u(h,h')p(h \mid s) \mid h \in H \}$ and he selects the hypothesis with the highest value of $E(h \mid s)$.

3.3 *Fluctuations* in p during test-runs are small (in probability). Fluctuations in u are always small (in probability).

3.4 If the optimal strategy does not involve randomization (which is the usual case) then assumptions 3.1, 3.2, and 3.3 predict that there is a $\phi : S \rightarrow H$, such that $n(s,h) \sim 0$ if $\phi(s) \neq h$.

4 *Assumptions: set II*

4.1 The subjects generates (during training) a *subjective probability distribution* p on S x H and a real *utility function* u on H x H.

4.2 If the utility function is regular then the subject responds $h \in H$ with probability $p(h | s)$ if presented with $s \in S$.

4.3 *Fluctuations* in p during test-runs are small (in probability). Fluctuations in u are always small (in probability).

4.4 If the utility function is regular then assumptions 4.1, 4.2, and 4.3 predict $n(s,h) \sim Np(s,h)$ for all $s \in S$ and $h \in H$.

5 *First hypotheses*

5.1 A simplification of the notation is possible if we number the stimuli as $s_1, \ldots, s_n$ and the hypotheses as $h_1, \ldots, h_m$. The data can now be written as a matrix n_{ij}.

5.2 With p_{ij} we mean the probability that the subject responds h_j if presented with s_i. According to section three p_{ij} is either zero or one, according to section four $p_{ij} = p(h_j | s_i)$.

5.3 The likelihood of the data is given by the multinomial distribution

$$p(n_{ij}) = \prod n_{i.}! \prod p_{ij}^{n_{ij}} / n_{ij}!$$

5.4 This makes it possible to test the hypothesis

$$H: p_{ij} = \pi(h_j | s_i)$$

by the usual multinomial BAN-methods (De Leeuw 1971a).

5.5 If $\psi : S \rightarrow H$ is the function defined by the optimality criterion 1.9 we can test the hypothesis

$$H: p_{ij} = \alpha \quad \text{if } \psi(s_i) = h_j, \quad p_{ij} = \beta \quad \text{if } \psi(s_i) \neq h_j$$

or one of the more general guessing correction hypotheses discussed in De Leeuw (1971b).

6 Freedom equations

6.1 Suppose there is a open subset Θ of Euclidean k-space and a $\theta \in \Theta$ and real valued functions f_{ij} on Θ such that $f_{ij}(\theta) = \pi(h_j | s_i)$.

6.2 Suppose moreover that for the function ψ defined in 5.5 there is a sequence θ_v in Θ such that $\lim_{v \to \infty} f_{ij}(\theta_v) = 1$ iff $\psi(s_i) = h_j$

6.3 It follows that the hypothesis H: $p_{ij} = f_{ij}(\theta)$ with $\theta \in \Theta$ has both the all-or-none strategy 3.2 and the event-matching strategy 4.2 as special cases (provided, of course, that $p = \pi$ and $u = \mu$).

6.4 In general the multinomial BAN theory allows for two types of testing. We can test the parametric specification $p_{ij} = f_{ij}(\theta)$ within the general multinomial model 5.3, and we can test (hierarchical) hypotheses about θ within this specification. Examples are given in De Leeuw (1971c), and in articles by De Klerk and/or Oppe on probabilistic functional learning and multinomial probabilistic concept learning which are being prepared for publication.

7 Constraint equations

7.1 Suppose there are functions f_k (k=1,..,v) on the set of all stochastic n x m matrices such that $f_k(\pi) = 0$ for all k, where π stands for the matrix $\pi(h_j | s_i)$.

7.2 Suppose moreover that $f_k(Q) = 0$ for all k if Q is the stochastic (binary) matrix defined by the function ψ of 5.5.

7.3 Again the 'objective' versions of the strategies 3.2 and 4.2 are special cases of the hypothesis H: $f_k(P) = 0$.

7.4 Again we can test within the hypothesis discussed in 7.3 by considering systems of equations implied by (but not equivalent to) $f_k(P) = 0$. Examples are again given in the references mentioned in 6.4.

8 References

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