MORE ON VARIANCE ESTIMATE

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1.

Suppose $\underline{a}_n$, $\underline{b}_n$, and $\underline{c}_n$ are three sequences of random vectors satisfying

$$\sqrt{n}\begin{bmatrix}\underline{a}_n-\alpha\\ \underline{b}_n-\beta\\ \underline{c}_n-\gamma\end{bmatrix}\overset{\mathcal{L}}{\Rightarrow}\mathcal{N}\left(\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}\Sigma_{aa}&\Sigma_{ab}&\Sigma_{ac}\\ \Sigma_{ba}&\Sigma_{bb}&\Sigma_{bc}\\ \Sigma_{ca}&\Sigma_{cb}&\Sigma_{cc}\end{bmatrix}\right)$$

We want to estimate a function $g(\beta)$, using an estimator of the form $f(a,b,c)$, with both f and g continuously differentiable.

For F-consistency we need $f(a,b,c)=g(b)$ for all a,b,c, and thus

$$\mathcal{D}_1 f(a,b,c)=0,$$
$$\mathcal{D}_2 f(a,b,c)=\mathcal{D}g(b),$$
$$\mathcal{D}_3 f(a,b,c)=0.$$

This implies

$$\sqrt{n}(f(\underline{a}_n,\underline{b}_n,\underline{c}_n)-g(\beta))\overset{\mathcal{L}}{\Rightarrow}\mathcal{N}(0,\omega),$$

with

$$\omega=\mathcal{D}g(\beta)'\Sigma_{bb}\mathcal{D}g(\beta)$$

Now suppose we know α. Then it suffices to require that $f(\alpha,b,c)=g(b)$ for all b and c. Thus

$$\mathcal{D}_2 f(\alpha,b,c)=\mathcal{D}g(b),$$
$$\mathcal{D}_3 f(\alpha,b,c)=0.$$

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This shows

$$\sqrt{n}(f(\underline{a}_n, \underline{b}_n, \underline{c}_n) - g(\beta)) \stackrel{\mathcal{L}}{\Rightarrow} \mathcal{N}(0, \nu),$$

with

$$\nu = \mathcal{D}_1 f(\alpha, \beta, \gamma)' \Sigma_{aa} \mathcal{D}_1 f(\alpha, \beta, \gamma) + \mathcal{D}g(\beta)' \Sigma_{bb} \mathcal{D}g(\beta) + \\ 2\mathcal{D}_1 f(\alpha, \beta, \gamma)' \Sigma_{ab} \mathcal{D}g(\beta).$$

Clearly

$$\nu \geq \nu_{min} \stackrel{\Delta}{=} \mathcal{D}g(\beta)' \{\Sigma_{bb} - \Sigma_{ba} \Sigma_{aa}^{-1} \Sigma_{ab}\} \mathcal{D}g(\beta),$$

with equality if and only if

$$\mathcal{D}_1 f(\alpha, \beta, \gamma) = -\Sigma_{aa}^{-1} \Sigma_{ab} \mathcal{D}g(\beta).$$

Moreover

$$\nu_{min} \leq \omega.$$

Example 1.1. Estimate the second moment μ_2 around the origin, using the first three sample moments m_1, m_2 and m_3 around the origin. We find

$$\omega = \mu_4 - \mu_2^2,$$

and if we know μ_1 we find

$$\nu_{min} = \mu_4 - \mu_2^2 - \frac{(\mu_3 - \mu_1 \mu_2)^2}{\mu_2 - \mu_1^2}$$

The minimum is attained for any f with

$$\mathcal{D}_1 f(\mu_1, \mu_2, \mu_3) = -\frac{\mu_3 - \mu_1 \mu_2}{\mu_2 - \mu_1^2},$$

$$\mathcal{D}_2 f(\mu_1, \mu_2, \mu_3) = 1,$$

for instance

$$f(m_1, m_2, m_3) = m_2 - \frac{m_3 - m_1 m_2}{m_2 - m_1^2}(m_1 - \mu_1)$$

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