book for the instructor teaching a course in probability. For example, the longer than usual treatments of infinitely divisible and stable laws (Sections 5.2 and 5.3) are valuable. Invariance principles (Section 5.4) and limit laws for a random number of random variables (Section 8.2) are rarely found in textbooks. These subjects merit inclusion in a graduate probability course because of their elegance.

Unfortunately the level of the book is too high and it is too mathematical (even more so than Billingsley 1979) to become a successful textbook. It is informative to compare Rao's book with Chung (1974). Chung's book has a "no frills" approach to the basic concepts. Each section is followed by numerous (533) timely exercises. A significant number of these exercises can be done by students without aid. In addition, after a new concept is introduced, a number of easy examples are included in the text. This makes Chung's book readable by students, an essential quality. For most students, Rao's book will need to be filtered through an instructor. The mathematical niceties and details, which are such delights to the instructor, will be forbidding to a student, since a person learning the subject does not understand which details can be safely overlooked. There are many institutions in the U.S. that use a sequence of two- or three-quarter courses to teach their statistics students probability *and* measure theory. In such courses a much shorter time is available to spend on probability theory. The aim, then, is to cover a number of basic concepts with relatively few details. Hence such a course cannot afford to do Markov and Martingale dependence (Chapter 3) before characteristic functions (Chapter 4) or the central limit theorem (Chapter 5). Moreover, applications must be done before Chapter 6. Topics can, of course, be selected out of order. But this can lead to unforeseen problems. Overall, I will undoubtedly use this book as a reference but will be very reluctant to use it as a textbook.

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Abstract Measurement Theory.

Louis Narens. Cambridge, MA: MIT Press, 1985. vii + 334 pp. $40.00.

What is abstract measurement theory? It is a mathematical discipline that studies conditions under which relational structures of a general type can be represented homorphically or isomorphically as relational structures on the real numbers. The prototype result is Hölder's theorem: an Archimedean totally ordered group is isomorphic to the reals (with the usual order, and with addition). This theorem has been extended in many directions, with great sophistication and surprising variety. Why? Because the correspondence that is established is said to define measurement. A set of weights and a pan balance can be used for the empirical definition of comparison and concatenation. We merely have to check whether our empirical results satisfy the axioms of an Archimedean totally ordered group. If this is the case, the weights can be represented by real numbers; that is, they are measured. There is another classical example, somewhat more interesting to readers of this journal. Starting with the work of Von Neumann and Morgenstern and of Savage, there have been many axiomatizations of the concepts of utility and probability. Again, the pattern of such theories from the mathematical point of view is to show that the axioms characterize some ordered mathematical structure that is isomorphic to a similar structure on the reals. If somebody's decisions satisfy the Savage axioms, then his acts will be represented by expected values for suitable utilities and probabilities.

In the early 1960s Luce, Suppes, Krantz, Tversky, and others started a major research program intended to show that numerical representation in any of the sciences was based on some such form of representation theorem. The book by Krantz, Luce, Suppes, and Tversky (1971) summarized most of the results obtained in the first 10 years. The book by Narens, discussed here, can be considered to be a more complete, more compact, and more abstract update of that earlier work. It is, by all accounts, a beautiful book. It requires considerable mathematical sophistication, or at the very least, a good working knowledge of modern abstract algebra. Some mathematical logic and set theory are also needed. The coverage of the area seems to be almost complete, and the treatment is both authoritative and elegant. Thus the book is good. But is it useful? And for whom?

To assess the usefulness of the book, we must point to some curious features of the area. Most of the papers on abstract measurement theory have appeared in the *Journal of Mathematical Psychology,* a perfectly respectable journal, but somewhat specialized and with a rather limited reach. This should be contrasted with the grandiose claims sometimes made by people active in this area. The methodological importance of abstract measurement theory has been overlooked by most philosophers of science, except Bunge (1973), who dismisses the results as irrelevant mathematical intrusions in the domain of natural science. Bunge ignores the fact that abstract measurement theory has inspired some interesting experiments, at least in psychology. It is clear, consequently, that aspects of measurement theory have relevance for theoretically minded psychologists. It is also clear that they do not have, or need to have, the mathematical sophistication required to read Narens's book. It could also be that some results presented in the book are a useful contribution to pure mathematics. I am not in a position to judge that. But more importantly for the readers of this journal, the results are not really of interest to most statisticians. Possible exceptions are, as previously indicated, people working in foundations with axiom systems for qualitative probability, independence, utility, and so on. It seems to me, however, that such philosophically oriented statisticians will benefit more from the content-oriented treatment by Krantz et al. (1971) and the more extensive treatment in various research papers by the same group.

In summary, this is a book of high quality. Reading it requires a great deal of intellectual effort but is quite rewarding from the aesthetic point of view. It seems to me that the practical value of the book is limited, certainly for statisticians and scientists dealing with real data. From a purely technical point of view, Narens improves on the work of Krantz et al. (1971) in many respects, but because the link with actual scientific practice is even thinner in this book, it will undoubtedly be much less influential.

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