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David J. Bartholomew, *Latent Variable Models and Factor Analysis*, London: Griffin, 1987, pp. 194.

I will start this review with a little story. In the fifties and sixties the Mathematical Center in Amsterdam was the center of academic statistics in the Netherlands. Of course there also was official statistics in the Netherlands Central Bureau of Statistics, and there were various forms of applied statistics in the departments of social and behavioral science, biology, and medicine. But the real statistics, the statistics that appeared in statistics journals, was the mathematical statistics of the Mathematical Center. In the early seventies things started to change a little bit. It became impossible to ignore the various developments in exploratory data analysis and in applied and computational statistics that were going on, for instance in psychometrics. There was, for instance, a technique called factor analysis, that seemed to have some popularity in those circles. The mathematical statistics group at the center decided to take a presumably definitive look at factor analysis. They did this by establishing a working group, that was generally known as study group on Suspect Methods. After a number of meetings the group published a long report (in Dutch) about the pitfalls of factor analysis. The main message of the report was that factor analysis techniques did not have the proper foundation in statistics that any data analysis technique needed. Moreover, the classical results of Guttman on determinacy of factor scores were discussed in great detail, as if they implied that there was something basically wrong with the technique. Nothing new was added to either the theory or practice of factor analysis.

Why this story? Because it can be used to put the book by Bartholomew, reviewed here, in the appropriate context. It makes it possible to indicate what has changed and what has remained the same in the attitude of statisticians towards factor analysis and related techniques. And in order to make my own position perfectly clear, I think that the idea of conditioning on latent elements to describe relations between variables in a simple way is the most important contribution of the social and behavioral sciences to data analysis. The fact that the methods do not always work well (the Spearman model does not fit intelligence tests, the Rasch model does not fit sets of

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correct-false items) is unfortunate, but does not diminish the value of the basic ideas.

Bartholomew seems to agree, and takes factor analysis seriously. It is no longer a suspect method. He states, in the preface, that factor analysis is now used quite frequently in geology, chemistry, sociology, and geography, and that the psychologist's approach to the technique may consequently be too narrow and too specific. This is undoubtedly correct. The classical psychological treatises on factor analysis concentrate on aspects which do not seem very relevant now (such as computation), on aspects which are of doubtful value (such as rotation) or which are completely irrelevant (such as empiricist or instrumentalist philosophy of science). Bartholomew's model is the classical text by Lawley and Maxwell (1963, 1971), although that text focuses almost exclusively on linear factor analysis, and still pays quite a deal of attention to computation.

According to Bartholomew "a principal tenet of the book is that the careful specification of a suitable probability model is the key to sound practice" (page vii). Am I mistaken, or do I vaguely hear the Mathematical Center again? It seems to me that for many applications of factor analysis and related techniques, the probability models, at least the inferential aspects of those models, are irrelevant. We approximate a covariance matrix or a multivariate cross table by a simple algebraic model, in order to smooth or to interpolate or to describe efficiently or even to understand our data. If there is a suitable sampling framework, it is unwise not to use it. If there is no such framework, it is unwise (indeed, it is suspect) to pretend that there is one.

The basic ideas behind latent variable models are due in all essentials to Fechner and Spearman, although it is easy to point to related ideas in the work of Galton, Pearson and Edgeworth, and although very similar ideas appeared later, quite independently, in other disciplines in the work of Wright, Frisch, and Kalman. Only fairly recently people have realized that Fechnerian psychophysics, econometric choice theory, the latent trait theories of item analysis, and the factor analysis of batteries of tests are really implementations of the basic ideas of conditional independence, given one or more latent variables. This synthetic point of view is due to Lazarsfeld and Guttman, with important early technical contributions by Lawley, and it was stated in full generality by Anderson (1959) and McDonald (1962).

Bartholomew's book is the first book that I have seen that consistently uses this approach. Conditional independence given the latent variable(s), also known as *local independence*, is the unifying idea. This is discussed, briefly and with considerable clarity, in Chapter 1. Section 1.5 is a (very disappointing) historical introduction, mainly to linear factor analysis. Again modeling is emphasized. "In a subject which has been criticized as arbitrary and hence too subjective, it is especially necessary to clarify in as rigorous a way as possible what is being assumed and what can be legitimately inferred'' (page 13-14). This does not address the question if these criticisms were appropriate in the first place, and it uses the word ''legitimately'' in a somewhat old-fashioned sense. Bartholomew dutifully regrets the fact that principal component analysis is often confused with factor analysis, but contributes to this confusion by discussing principal component analysis in various places in a book in which it obviously does not belong.

The chapters of the book follow a predictable pattern. Latent variables can be discrete, which defines latent class models, discussed in Chapter 2. If the latent variables are discrete, the observed or manifest variables can be either binary, polytomous, or continuous. There are sections 2.2, 2.3, and 2.4. They are clear, but short. Chapter 3, with continuous latent and observed variables, (too) briefly discusses linear factor analysis. The likelihood equations, maximum likelihood estimates, chi square tests, standard errors, EM algorithm, scale invariance, factor score indeterminacy, Heywood cases, asymptotically distribution free methods, confirmatory factor analysis, bootstrap methods, and rotation are discussed in about 30 pages. This leads to a treatment which is not exactly balanced. The usual tedious and uninteresting manipulations of the likelihood equations are outlined in detail. Even Rao's antiquated algorithm is discussed. There are many useful references, mainly to recent literature, sometimes a bit arbitrarily chosen. The practical advice is often sound, but the contributions of numerous authors are summarized in single sentences. Again the historical introductions to the various sections are quite abysmal. The first systematic treatment of latent class analysis was by Lazarsfeld in the early fifties, not by Lazarsfeld and Henry (1968). Latent profile analysis is due to Gibson and Anderson, not to Lazarsfeld and Henry. The method of moments has not been superseded by the maximum likelihood approach, although maybe a lot of statisticians think so (because it says so in their textbooks). Swain did not show what is attributed to him on page 48. The term ''zig-zag routine'' (page 46) was already commonplace in mathematical programming when Magnus and Neudecker were still students.

The original part of the book, summarizing some of Bartholomew's own work, starts in Chapter 4. The idea of local independence is combined there with the classical Koopmans-Barankin theory characterizing exponential families in terms of existence of sufficient statistics. Another interesting unifying idea is the use of ''posterior expectations'' to compute factor scores and positions in the latent space in general. This is, again, not placed in the proper historical context. The use of posterior distributions in this context (although not necessarily under that unfortunate name) was already advocated in the context of factor analysis by Thurstone and Bartlett, and in the context of latent class analysis by Lazarsfeld. In the related area of variance components analysis it is commonplace.

The next four chapters (5: Models of binary data; 6: Methods for binary data; 7: Models of polytomous data; 8: Methods for polytomous data) also contain much of Bartholomew's own work in the area. Two key themes are the formulation of models as either Response Function Models or Underlying Variable Models, and the use of approximations to the likelihood functions. The Response Function Models, also known as latent trait models, directly model the dependency of the categorical response and the position on the latent continuum by assuming a logit or probit regression. The Underlying Variable Models postulate an underlying normal process, and then model the categorical responses as discretized versions of the continuous latent responses. Under very general conditions the two models are equivalent. This has been noted many times before, first perhaps in utility and probabilistic choice theory, later in test theory. We shall not review the contents of these four chapters in detail. They are much like the rest of the book, competent, well-done, clear, concise. But they also give the impression of having been done in relative isolation, sometimes not giving credits where they are due. Treatment of computational aspects is thorough, with a lot of welcome attention paid to the EM algorithm. The approximations, based on correlations, are useful, not so much because they should really be used routinely, but because they show the limitations of using correlation or cross-products methods in this context, and the types of distortions one can expect. Chapter 9 has some useful examples, but of course collecting them in a separate chapter already indicates that they are not really well-integrated with the text. There is very little interpretation of the examples, but the computations and the statistics are illustrated nicely.

The conclusion of this review is that Bartholomew's book is very good and useful, especially for statisticians who do not know about latent variable theory, and perhaps also for sophisticated users of the methods. Bartholomew's own contributions to the area have been very useful, and they are nicely summarized here. For experts in psychometrics or choice theory modeling the book has some irritating aspects, mainly because it is quite bad from a historical and systematic point of view. Nevertheless there is no book quite like it, and its basic approach is both beautiful and sound. It is unfortunate, but significant, that no psychometrician has found the time and the energy to write a book like this about five years ago, when all necessary material was already there. As in multidimensional scaling and cluster analysis, the really interesting books in our field either do not exist, or are written by statisticians.

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