A Two-stage Procedure for Analyzing a Brand Switching Matrix

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Overview

We propose a simple two-stage procedure to analyze the 15 x 15 1989 French car switching matrix. First, we correct for the asymmetry in the matrix. This modeling effort produces asymmetry-corrected dissimilarity measures among automobiles that behave like distances. Also obtained at this stage are marginal popularity measures for each automobile. We then scale the dissimilarities with a restricted multidimensional scaling model that introduces an extra parameter for each brand, interpretable in terms of brand loyalty. The results offer interesting managerial insights into brand switching.

Models

We consider brand switching as possessing both symmetric and asymmetric components. Symmetric switching is further decomposed into an aspect due to similarity among the automobiles and an aspect due to brand loyalty. Similar ideas, but with explicit emphasis on graphically modeling the asymmetric components of brand switching, are developed in Hoffman and van der Heijden (1991).

We begin with the general idea of correcting for any asymmetry in the table by applying Luce's choice model (1963):

$$\pi_{ij} = \alpha_i \beta_j \eta_{ij} \tag{1}$$

with the η_{ij} symmetric, i.e. $\eta_{ij} = \eta_{ji}$. Note that this is identical to the quasi-symmetry model, due to Caussinus (1965), except that quasi-symmetry uses the usual ANOVA constraints on the log scale while in the choice model, α is identified by the fact that the data are row-normalized (rows add to one) and scaled in such a way that $\eta_{ii} = 1$, for all *i*. Thus, we must have:

$$\sum_k \pi_k = \alpha_i \sum_k \beta_k \eta_{ik} = 1, \qquad k = 1, \ldots, m$$

$$\alpha_i = 1 / \Sigma_k \eta_{ik} \beta_k$$

and thus,

$$\pi_{j|i} = \frac{\beta_j \eta_{ij}}{\sum_k \beta_k \eta_{ik}} \tag{2}$$

which is the usual choice model formulation, with $\pi_{j|i}$ the probability of switching to automobile *j* from automobile *i*.

Expression (2) models the proportion who switch from automobile *i* to automobile *j* on the basis of the perceptual similarity of *i* and *j* (η_{ij} - the perceptual parameters) and characteristics of the cars (β_j - the response bias parameters). Takane and Shibayama (1986) call (2) the unconstrained similarity-choice model.

In the psychophysical context, β_j is also referred to as the motivational parameter and assumed to depend on such stimulus factors as frequency of presentation. Holding perceived similarity of a pair of stimuli constant, the model predicts that a stimulus more familiar to the respondent (higher β_j) will be chosen more often than a less familiar stimulus. Thus, in our brand switching framework, β_j measures the bias arising from market share effects and renders an interpretation in terms of marginal popularity. The β_j's thus indicate the extent of asymmetry in choice for each automobile.

Consider first, from (1), that taking logs of both sides gives

$$\ln \pi_{ij} = \ln \eta_{ij} + \ln \beta_j + \ln \alpha_i \qquad (3)$$

The first term on the right hand side of (3) gives the symmetric component of choice and the second term gives the asymmetric component. In this model, the asymmetric aspect of switching depends only on β_j. It is easily verified that if all automobiles are perfectly similar, (i.e. $\eta_{ij} = 1$ for all *i* and *j*), $\pi_{ij} = \beta_j/\Sigma_k\beta_k = \beta_j$, (since $\Sigma \ln \beta_k = 0$) the probability of switching from automobile i to automobile j is determined solely by the marginal popularity of automobile j.

The η_{ij} can be interpreted as dissimilarity measures. We can assume in addition (Luce 1963) that they are distances between vectors in multidimensional space:

$$-\ln \eta_{ij} \;=\; d_{ij}(X) \qquad (4)$$

where X is the m'p matrix of coordinates for each automobile in p-dimensional space.

The second stage of our proposal involves relating the dissimilarities in (4) to a distance model:

$$-\ln \eta_{ij} \;=\; (d^2{}_{ij}(Z) + \delta_i^2 + \delta_j^2)^{½} \qquad (5)$$

Here, we introduce extra uniqueness parameters to capture automobile brand loyalty. This means that X is now of the form X = **ZΔ,** with Δ diagonal; X now has p+m dimensions. This restricted multidimensional scaling model (i.e. a distance model with uniquenesses) is discussed in Bentler and Weeks (1978), de Leeuw and Heiser (1980), and Winsberg and Carroll (1989). If all automobiles are equally similar to each other, then the δ^2 parameters completely determine dissimilarity.

The map constructed from $d_{ij}(X)$ in (5) displays the similarity aspect of symmetric switching, so that automobiles near each other are more likely to switch into each other. Thus, this map reveals a form of market structure. Takane and Shibayama (1986) discuss several interesting models which impose additional structure on the η_{ij}.

The remaining component of symmetric switching is attributable to the diagonal in the switching matrix and represented by Δ. Thus, the uniquenesses give the proportion of variance not accounted for by the similarity space. The larger the uniqueness, the greater the degree to which switching is not determined by the similarity space, but by characteristics unique to that brand, interpretable in terms of brand loyalty. We draw an analogy with the common factor model, with uniquenesses $1- h_j^2$, where h_j^2 denote the commonalities.

Implementation

We implement our two-stage procedure by first fitting the choice model via iterative proportional fitting (Bishop, Fienberg, and Holland 1975). Then, we fit the distances according to the SMACOF algorithm outlined in de Leeuw and Heiser (1977; 1980).

Results

The likelihood ratio chi-square (G2) for the choice model in (2) equals 115 with 91 degrees of freedom (p=.115), so that the model fits. The values in Table 1 give the log of marginal popularity. Higher values indicate greater degrees of asymmetry. Forming the difference $\log \beta_i^2 - \log \beta_j^2$ gives an indication of the relative attractiveness of the (*i,j*) pair to consumers (i.e. the strength and direction of asymmetry). For example, Renault to Mercedes = 328, while Mercedes to Renault = -328. The estimates in Table 1 comprise a scale of asymmetry associated with the automobiles switched into; the scale reveals that the more popular cars tend to be sporty, foreign and expensive.

Table 1
Asymmetry (ln β) estimates from the Choice Model

Brand	ln β
Alfa Romeo	12.4053
BMW	5.9880
Citroen	-9.6597
Fiat	-5.9344
Ford	-8.8489
GM	-6.7939
Lada	9.4777
Mercedes	2.0724
Peugeot	-16.5243
Renault	-18.2424
Rover	6.1171
Saab	21.5875
Seat	8.9428
VW/Audi	-9.7702
Volvo	9.1829

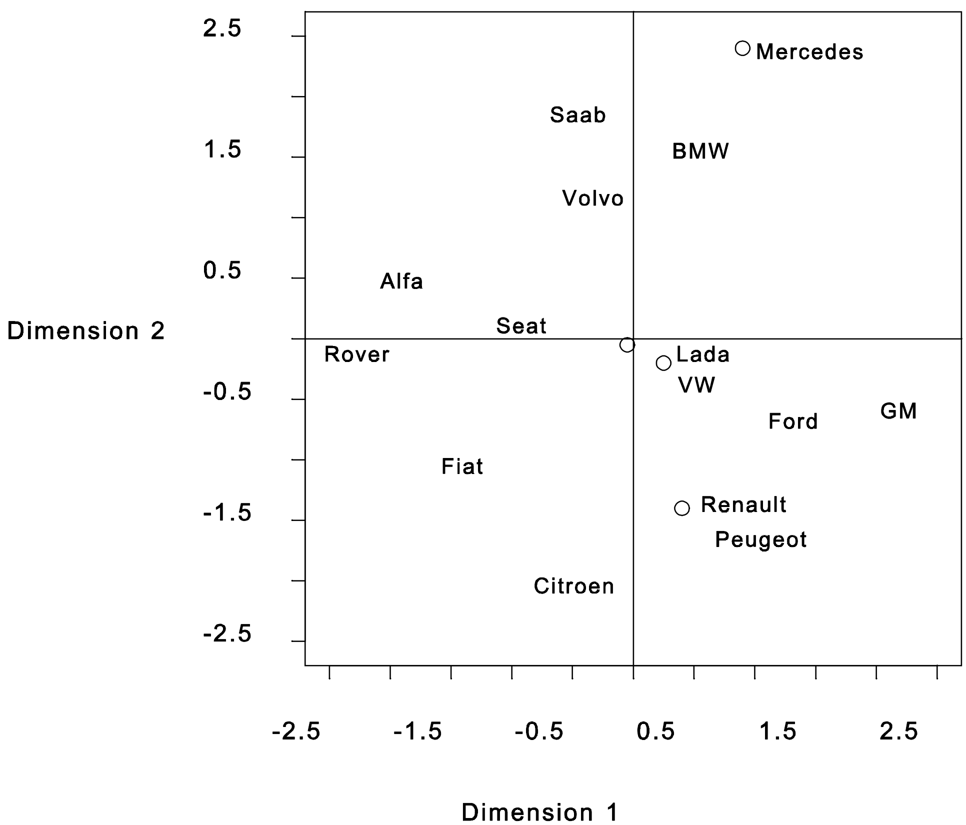
The map in Figure 1 shows the restricted MDS solution based on the estimated dissimilarities (RMS error=.11). The dissimilarities are shown in Table 3 and the fitted values in Table 4. We interpret the map as follows. Dimension one is characterized by Alfa Romeo on the one hand, and American automobiles, notably GM and Ford, on the other. The relative positions of the other cars suggest that dimension one contrasts sporty cars with plain cars. Although we point out that dimension one can also be viewed as a contrast between British and American cars. Expensive Swedish and German luxury automobiles define the positive direction of dimension two, with expensive French and Italian cars having negative scores on this dimension. Thus, dimension two also appears to distinguish cars on the basis of nationality. Additionally, smaller cars tend toward the center of the map, with the less expensive, more plain cars on the right, and the fancy, more expensive cars on the left.

Table 2
Brand Loyalty Estimates from the MDS Model

Brand	Brand Loyalty
Alfa Romeo	.0667
BMW	1.3818
Citroen	.2321
Fiat	1.1983
Ford	1.3002
GM	.1289
Lada	2.0588
Mercedes	.0754
Peugeot	.7724
Renault	.8512
Rover	1.5728
Saab	.3041
Seat	1.1345
VW/Audi	1.4946
Volvo	1.6369

The map depicts symmetric switching on the basis of perceptual similarity among the automobiles. The position of Alfa Romeo, for example, suggests consumers view it as a sporty alternative to the larger, more expensive luxury automobiles. However, the dominant feature in these data suggests switching on the basis of automobile country of origin. This appears reasonable considering the level of aggregation of the data.

The scaling uniquenesses shown in Table 2 are interpreted as estimates of brand loyalty in the context of symmetry. The idea is that each automobile has a score on its own unique dimension, in addition to its position in multidimensional similarity space. Each uniqueness score represents the variance in switching unaccounted for by the similarity space; thus, the uniquenesses indicate a type of brand loyalty for each automobile. Notice that the pattern of brand loyalty is quite different from the pattern of asymmetry. So, for example, switching for Alfa Romeo and Mercedes is almost completely explained by their positions in the map in Figure 1. In contrast, even after taking symmetric switching into account, brand loyalty is an important component of choice for Lada, Rover, VW, and Volvo. Observe that cars near the origin tend to have large uniquenesses.



Conclusion

Our two-stage approach to modeling a brand switching matrix yields a scale of asymmetry for the 15 automobiles, a market structure map showing symmetric switching among the cars, and measures of uniqueness interpretable in terms of brand loyalty. These distinct, yet complementary results give interesting managerial insights into brand switching among consumers.

Table 3

Dissimilarities (-ln η)

0.00	2.92	3.34	2.65	3.23	3.49	3.58	3.29	3.21	3.12	2.95	2.40	2.48	3.09	2.94
2.92	0.00	3.37	3.46	3.19	3.41	3.78	2.06	3.04	3.09	2.94	2.24	3.01	2.60	2.97
3.34	3.37	0.00	2.30	2.81	3.05	2.99	3.90	1.85	1.97	2.93	4.00	2.66	2.88	3.43
2.65	3.46	2.30	0.00	2.42	2.46	2.55	3.90	2.12	1.93	2.17	3.15	2.24	2.23	3.32
3.23	3.19	2.81	2.42	0.00	2.06	2.92	3.55	2.21	2.14	2.35	3.44	2.19	2.43	3.40
3.49	3.41	3.05	2.46	2.06	0.00	2.63	3.72	2.22	2.20	3.36	3.32	2.14	2.35	3.18
3.58	3.78	2.99	2.55	2.92	2.63	0.00	3.69	2.66	2.99	2.50	2.48	2.62	3.51	3.41
3.29	2.06	3.90	3.90	3.55	3.72	3.69	0.00	3.72	3.62	3.36	2.35	3.21	3.49	2.98
3.21	3.04	1.85	2.12	2.21	2.22	2.66	3.72	0.00	1.54	2.67	4.02	2.30	2.09	3.36
3.12	3.09	1.97	1.93	2.14	2.20	2.99	3.62	1.54	0.00	2.50	4.06	2.00	2.26	3.33
2.95	2.94	2.93	2.17	2.35	3.36	2.50	3.36	2.67	2.50	0.00	2.73	2.33	2.35	2.97
2.40	2.24	4.00	3.15	3.44	3.32	2.48	2.35	4.02	4.06	2.73	0.00	2.21	2.98	2.08
2.48	3.01	2.66	2.24	2.19	2.14	2.62	3.21	2.30	2.00	2.33	2.21	0.00	2.12	2.59
3.09	2.60	2.88	2.23	2.43	2.35	3.51	3.49	2.09	2.26	2.35	2.98	2.12	0.00	2.76
2.94	2.97	3.43	3.32	3.40	3.18	3.41	2.98	3.36	3.33	2.97	2.08	2.59	2.76	0.00

Table 4

Fitted Distances

0.00	2.99	2.81	2.56	3.61	4.18	3.25	3.68	3.40	3.31	2.41	1.99	2.48	2.96	2.33
2.99	0.00	4.03	3.22	3.01	3.03	3.06	1.79	3.49	3.42	2.77	1.83	2.43	2.72	2.36
2.81	4.03	0.00	1.66	2.71	3.17	2.99	4.86	1.67	1.69	2.54	3.98	2.45	2.63	3.63
2.56	3.22	1.66	0.00	2.32	2.66	2.60	3.91	1.77	1.75	2.17	3.21	1.94	2.19	2.99
3.61	3.01	2.71	2.32	0.00	1.57	2.58	3.37	1.84	1.85	2.56	3.45	2.09	2.15	3.25
4.18	3.03	3.17	2.66	1.57	0.00	2.65	3.18	1.89	1.93	2.85	3.72	2.28	2.21	3.51
3.25	3.06	2.99	2.60	2.58	2.65	0.00	3.45	2.57	2.54	2.68	3.19	2.37	2.54	3.13
3.68	1.79	4.86	3.91	3.37	3.18	3.45	0.00	4.10	4.03	3.31	1.89	2.93	3.15	2.62
3.40	3.49	1.67	1.77	1.84	1.89	2.57	4.10	0.00	1.16	2.43	3.80	2.07	2.13	3.46
3.31	3.42	1.69	1.75	1.85	1.93	2.54	4.03	1.16	0.00	2.38	3.71	2.02	2.10	3.39
2.41	2.77	2.54	2.17	2.56	2.85	2.68	3.31	2.43	2.38	0.00	2.63	1.98	2.29	2.63
1.99	1.83	3.98	3.21	3.45	3.72	3.19	1.89	3.80	3.71	2.63	0.00	2.46	2.89	1.83
2.48	2.43	2.45	1.94	2.09	2.28	2.37	2.93	2.07	2.02	1.98	2.46	0.00	1.91	2.44
2.96	2.72	2.63	2.19	2.15	2.21	2.54	3.15	2.13	2.10	2.29	2.89	1.91	0.00	2.81
2.33	2.36	3.63	2.99	3.25	3.51	3.13	2.62	3.46	3.39	2.63	1.83	2.44	2.81	0.00