

Component and Correspondence Analysis

Dimension Reduction by Functional
Approximation

Edited by

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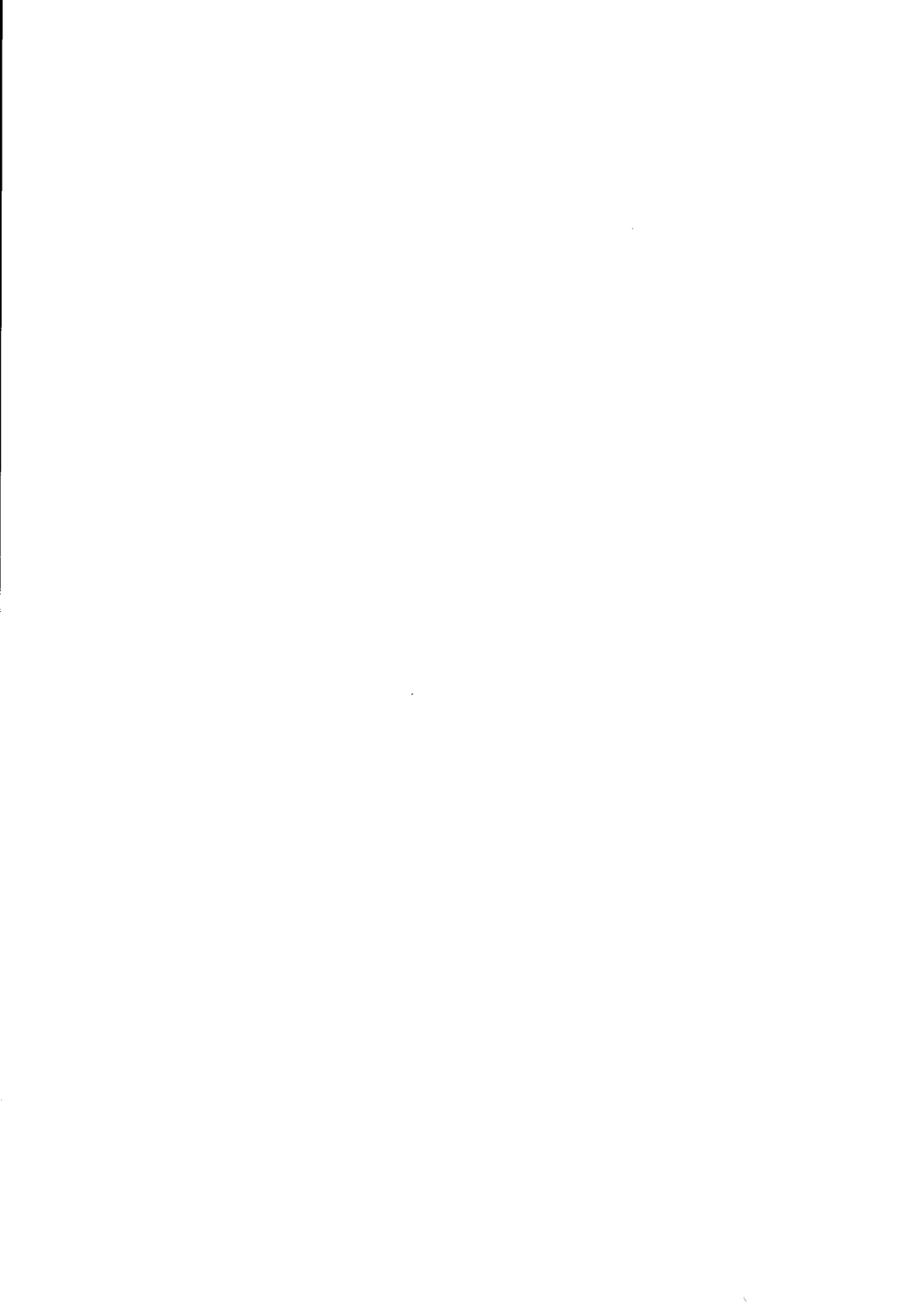
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